Supercritical Dirac states: an update

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Lisbon, September 2009

1) supercritcal Dirac: square well example

2) resonance calculations by analytic continuation

3) differential equation approach

4) U-U collisions - what will we see ?

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NSERC CRSNG People. Discovery. Innovation.

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DAAD Deutscher Akademischer Austausch Dienst German Academic Exchange Service

3d square well in Dirac QM

free particle in a box ?   W.Greiner: RQM, Exercise 9.5

$$\frac{d g_\kappa}{d r} + \frac{\kappa}{r} g_\kappa = (E + m_\mathrm{e}) f_\kappa - V(r)\, f_\kappa \quad , \qquad (1)$$

$$\frac{d f_\kappa}{d r} - \frac{\kappa}{r} f_\kappa = -(E - m_\mathrm{e}) g_\kappa + V(r)\, g_\kappa \quad . \qquad (2)$$

Units: $\hbar = 1, c = 1$ from now also: $m_\mathrm{e} = 1$

Angular momentum label: $\kappa = -1$ yields $j = 1/2$

g/f components - for $V(r) = V_0$ combine+understand

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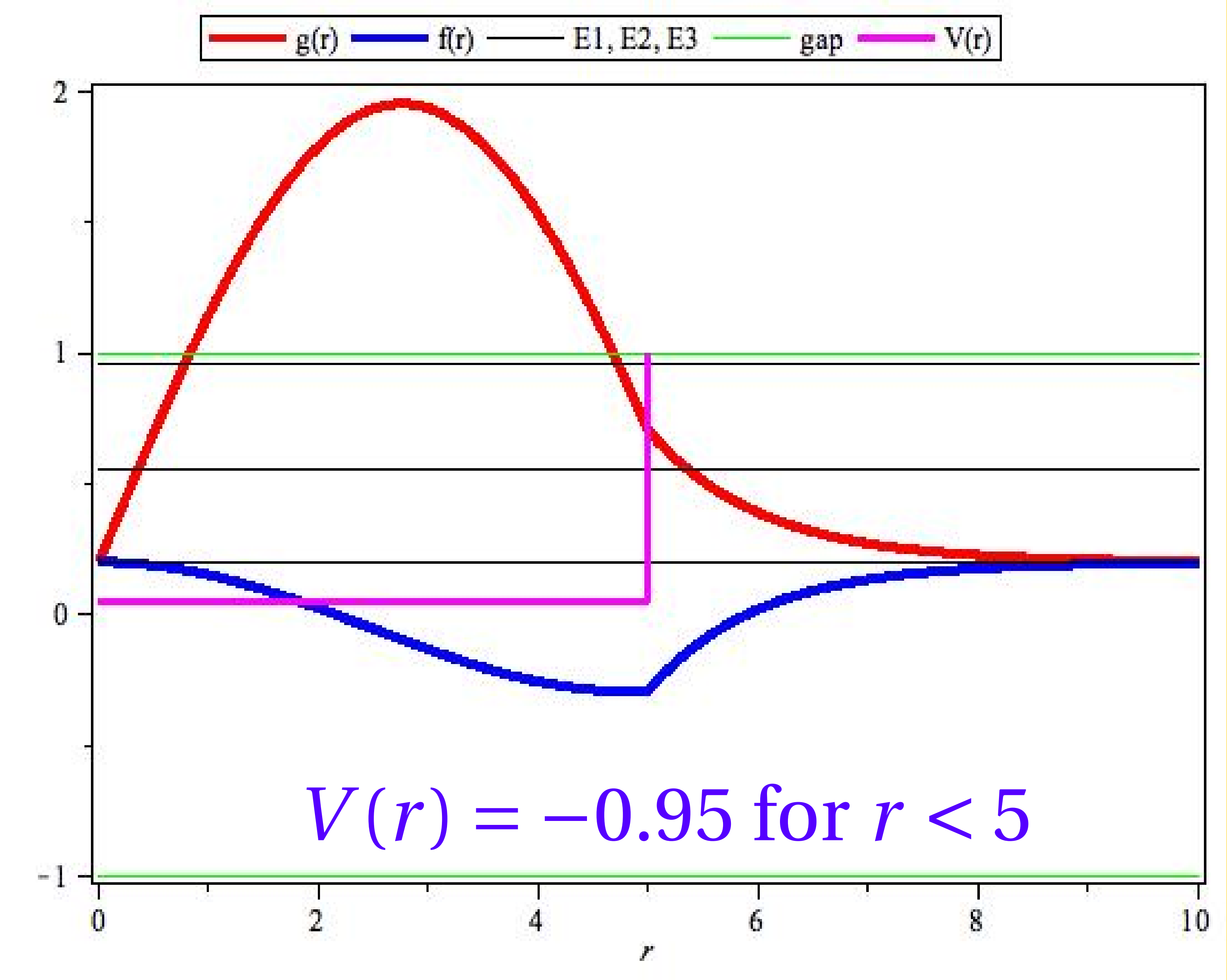
$V(r)$: vector coupling vs scalar (mass) coupling

Potential step with $V_0 < 1$

large zero-point E

just 3 states

kinks ?



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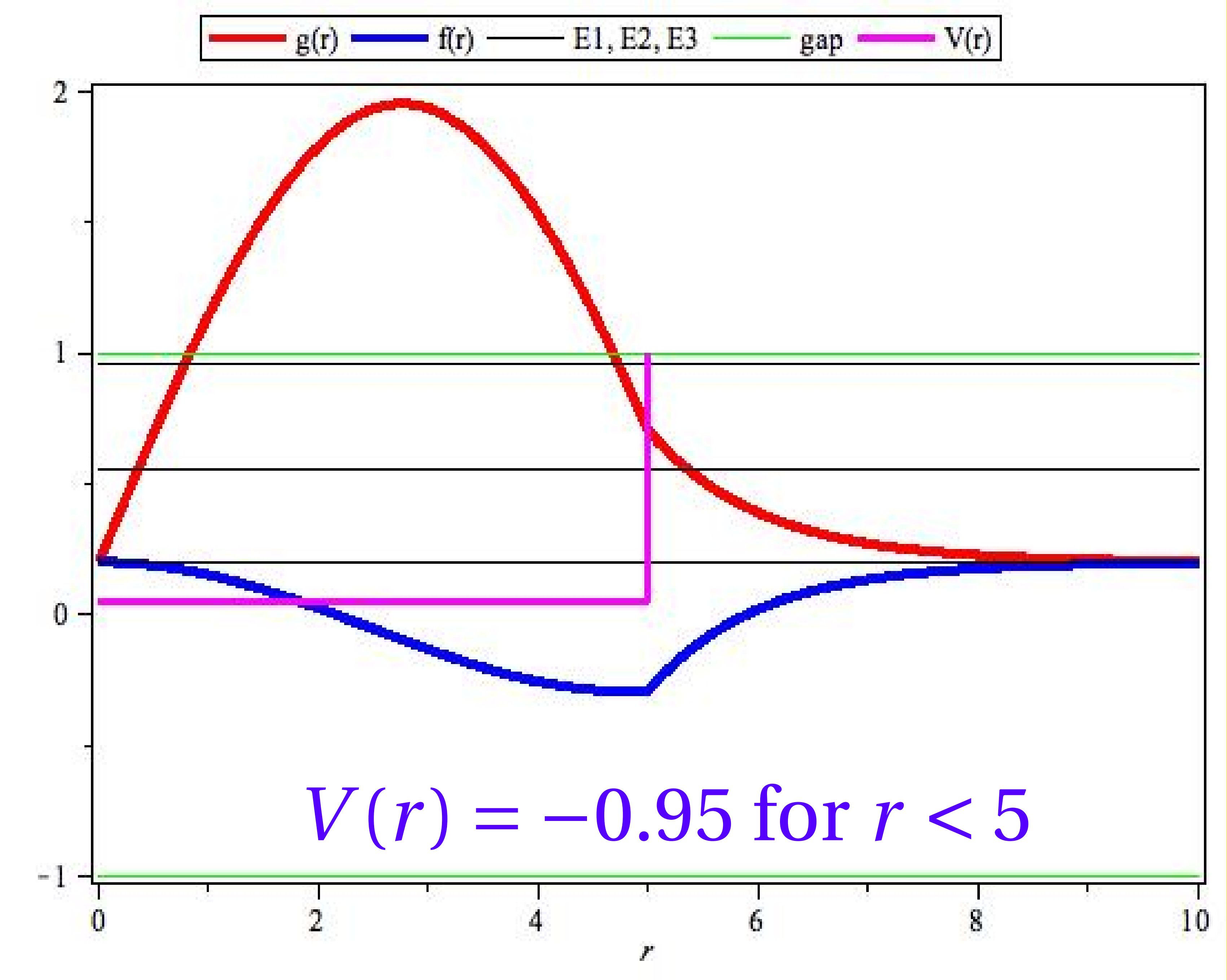

deeply bound QM particle squeezed to $5\lambda_C$

lower component has a centrifugal barrier

Giant potential step $\Delta V > 2$

$V(r) = -0.5$ for $r < 20$ and $V(r) = 1.65$ for $r > 20$

Using a trick obtain a resonance solution:

$E \approx 0.5123 - 0.000153i$

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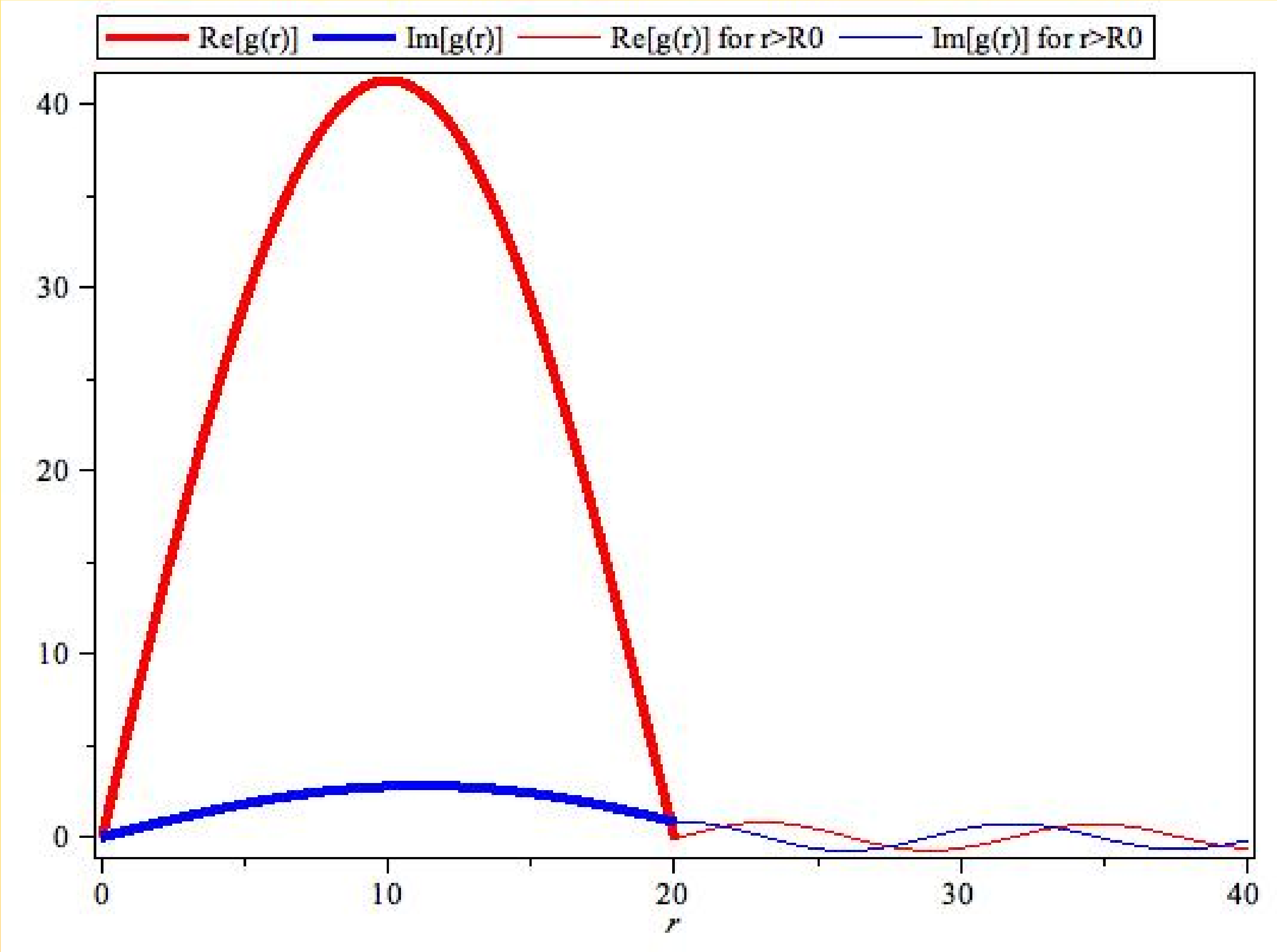

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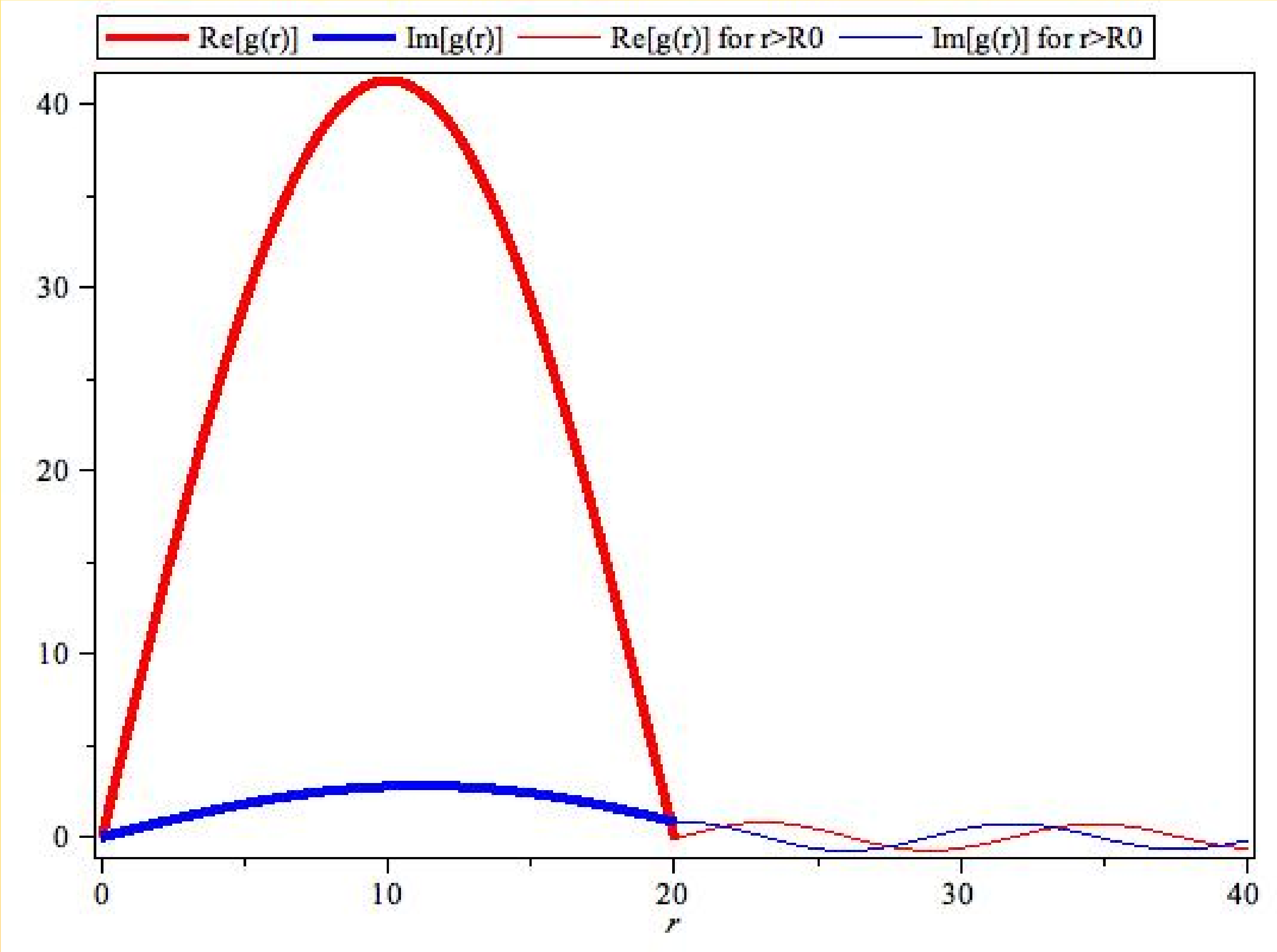

smaller zero-point E

barrier region ?

why embedded in continuous E ?

Asymptotic region has a mass gap shifted up by $\Delta E = 1.65$

$E_{\text{res}} \approx 0.51$ connects with $E < -mc^2$ states

What does it mean?

Dirac sea interpretation: $E < -mc^2$ states all occupied

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e^+ distribution when we look at times $t << \tau$?

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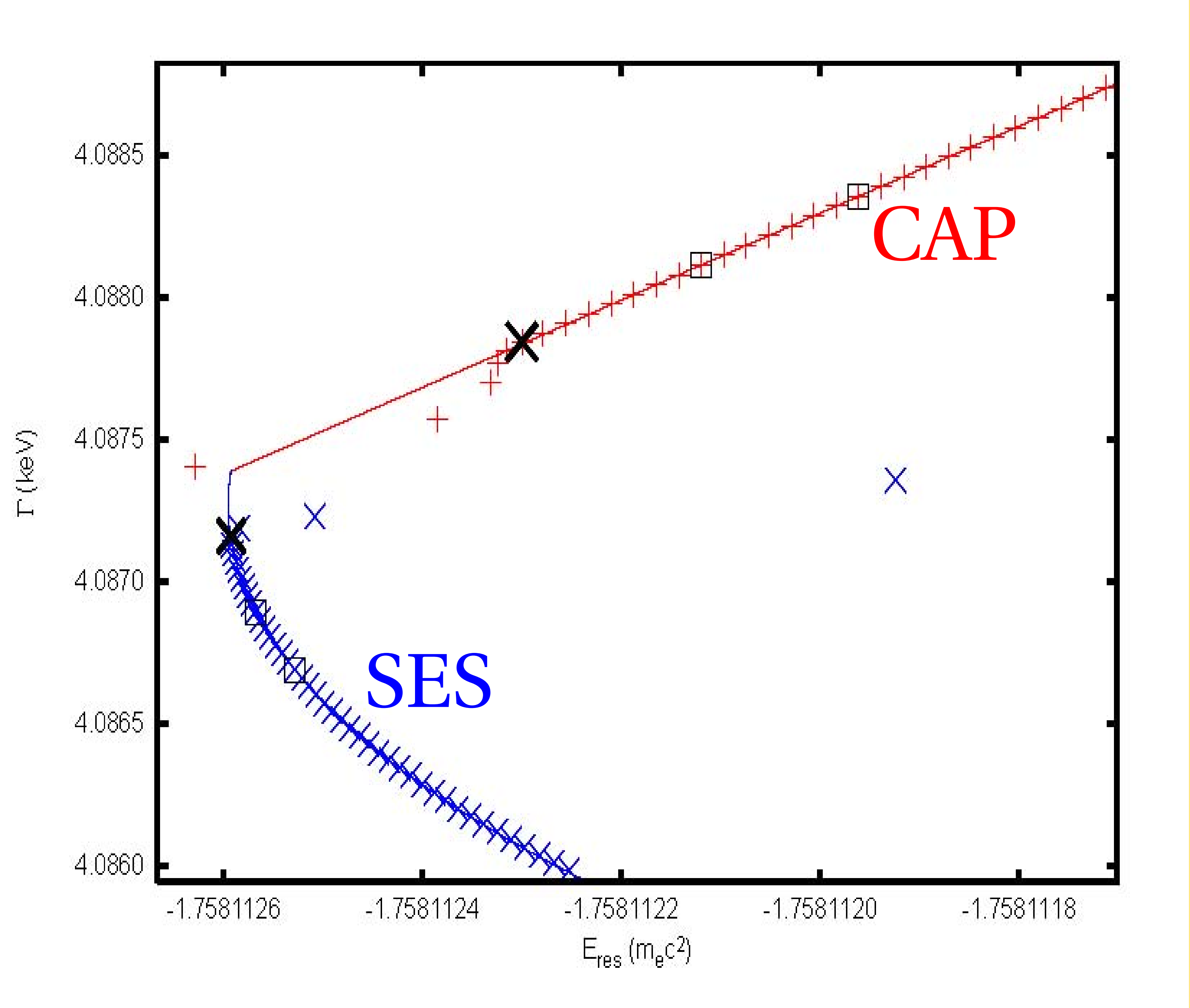
Stabilization: least sensitivity to scaling parameter

Extrapolation to zero imaginary stuff:

compute parametric dependence of $E_{\text{res}} + i\Gamma/2$

observe complex trajectory, Pade approximate

Example: PRA 76, 022503 $_{92}U^{238}$ - $_{98}Cf^{251}$ at 20 fm



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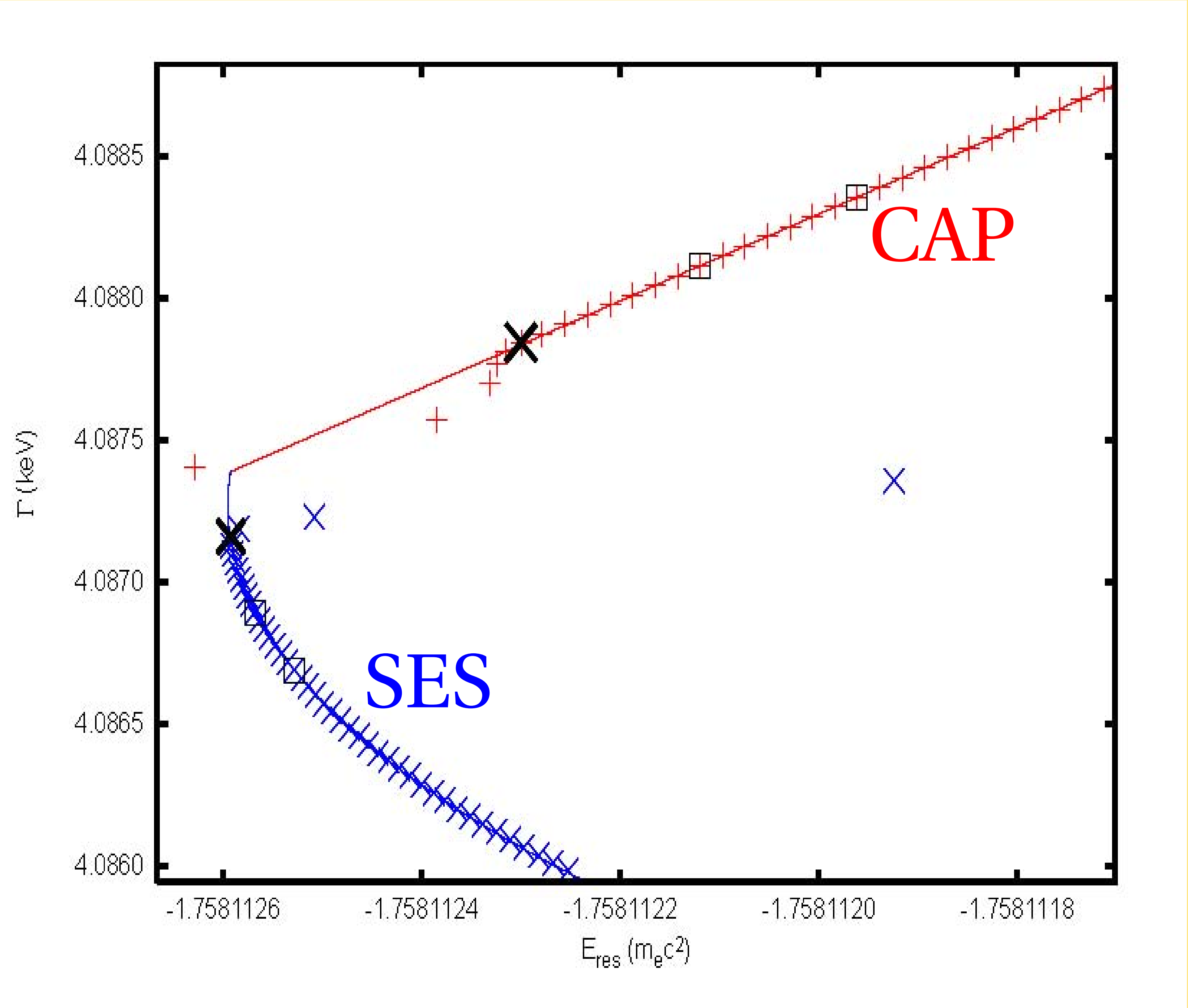

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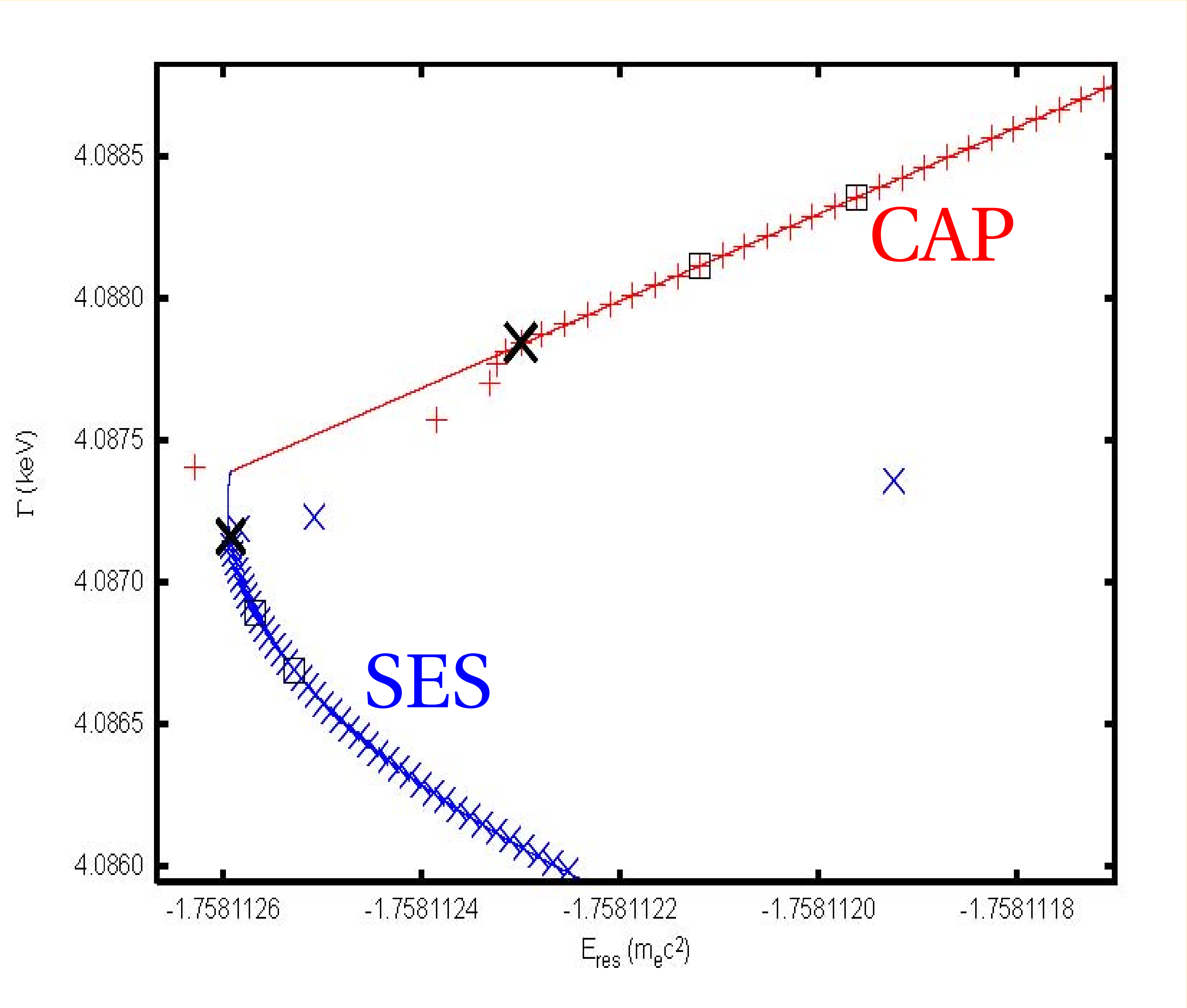

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Finite basis can't cope with large-*r* demand to represent the oscillatory tail (with minimal attenuation)

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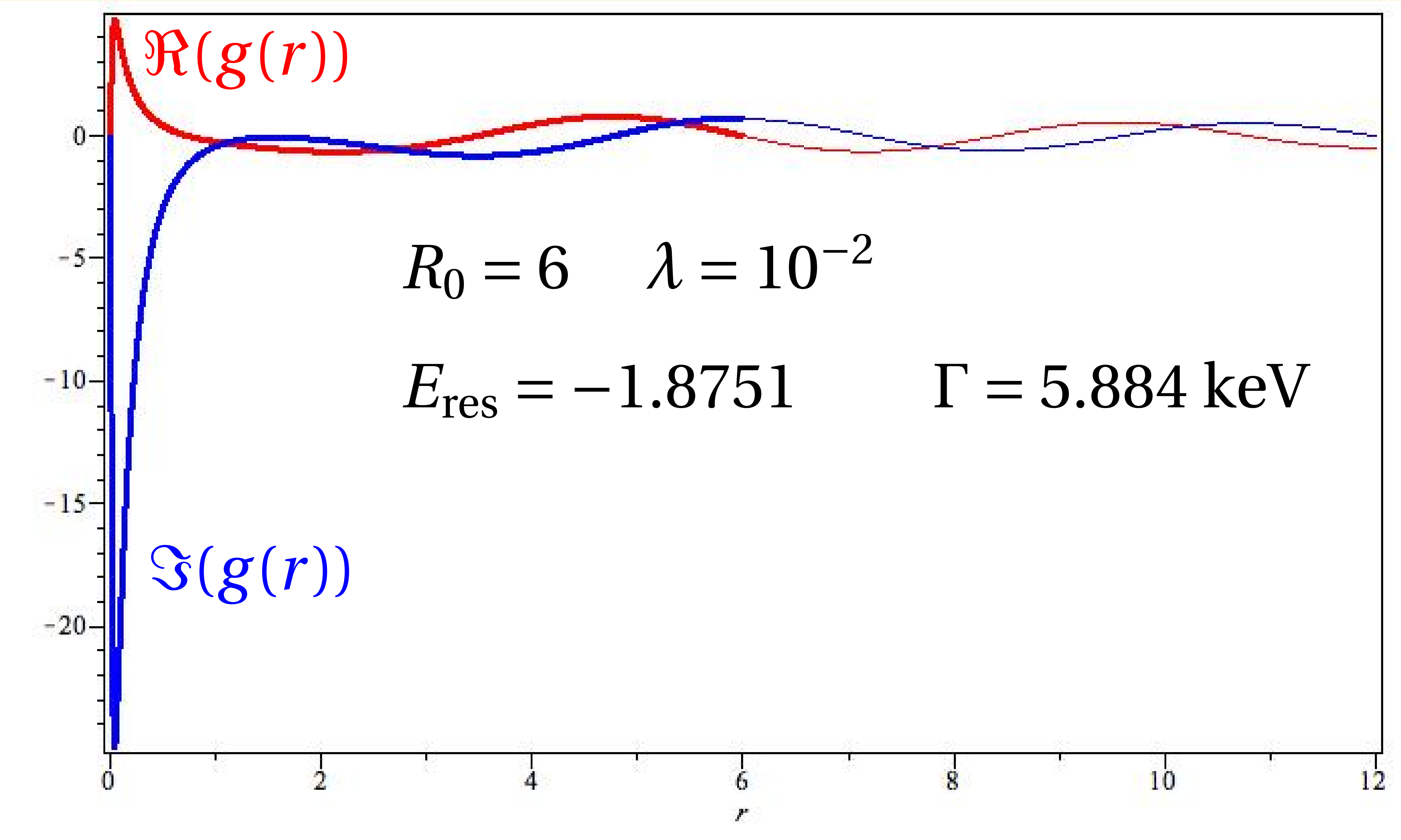
Implement multipole expanded $V(r,R)$

Supplement the mass terms with a scalar CAP

$$V_{\mathrm{CAP}} = \lambda (r-3)^2 i \quad \text{for} \;\; r > 3$$

U-Cf CM frame, $R = 20$ fm [$\kappa = -1$ only]

Introduce cut-off R_0; for $r > R_0$: $V(r) = V(R_0)$ and $V_{\mathrm{CAP}}(r) = V_{\mathrm{CAP}}(R_0)$



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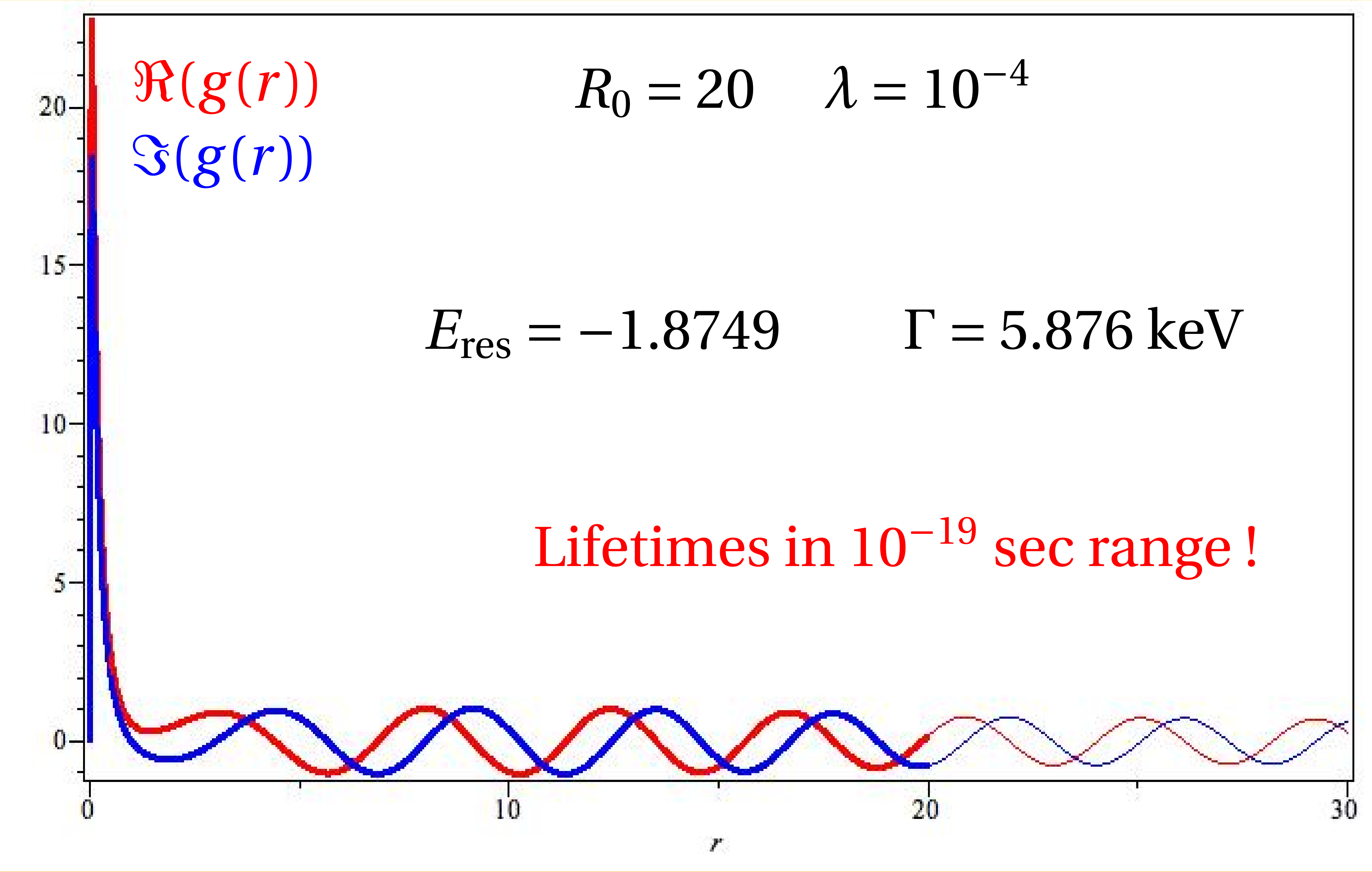

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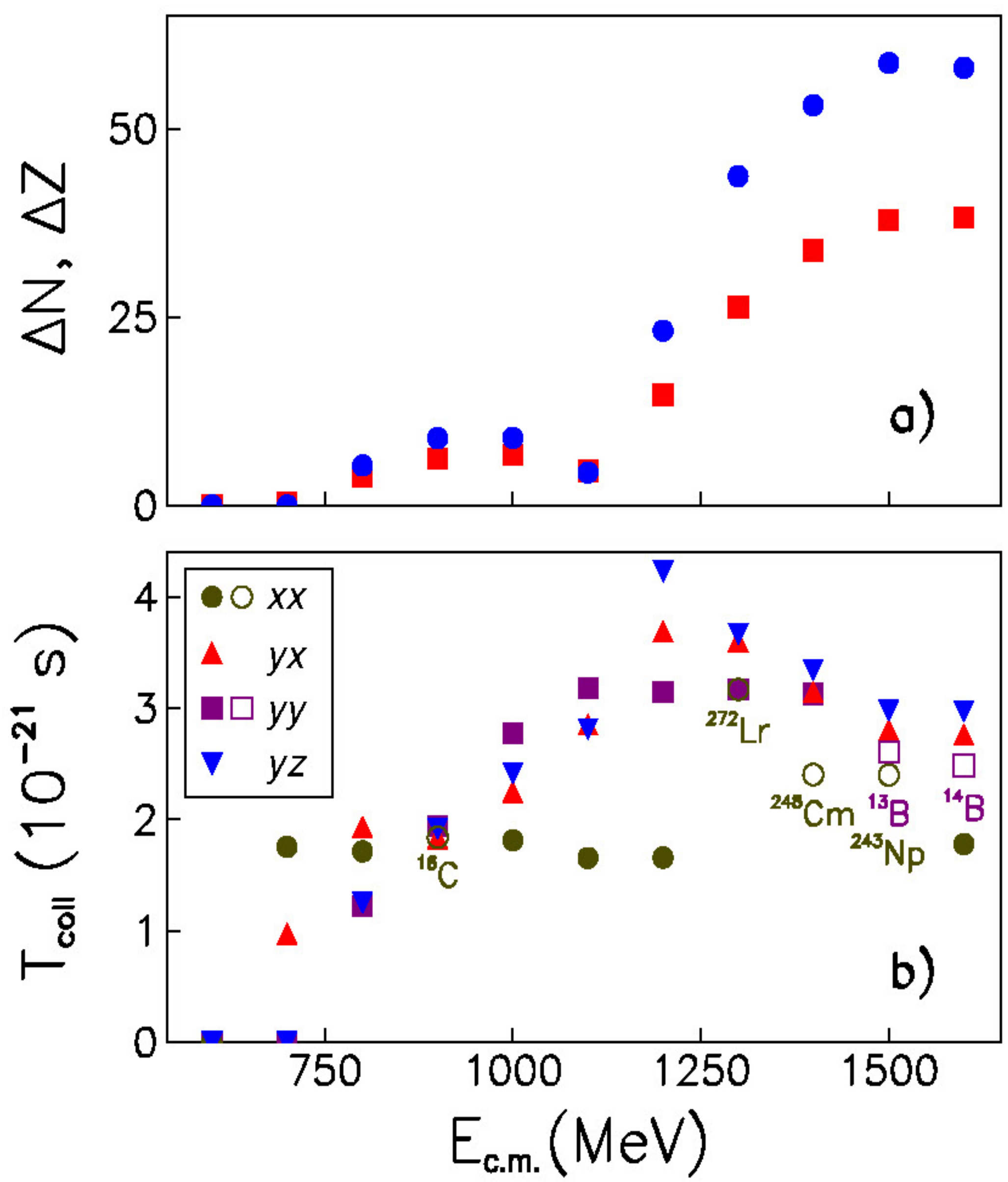

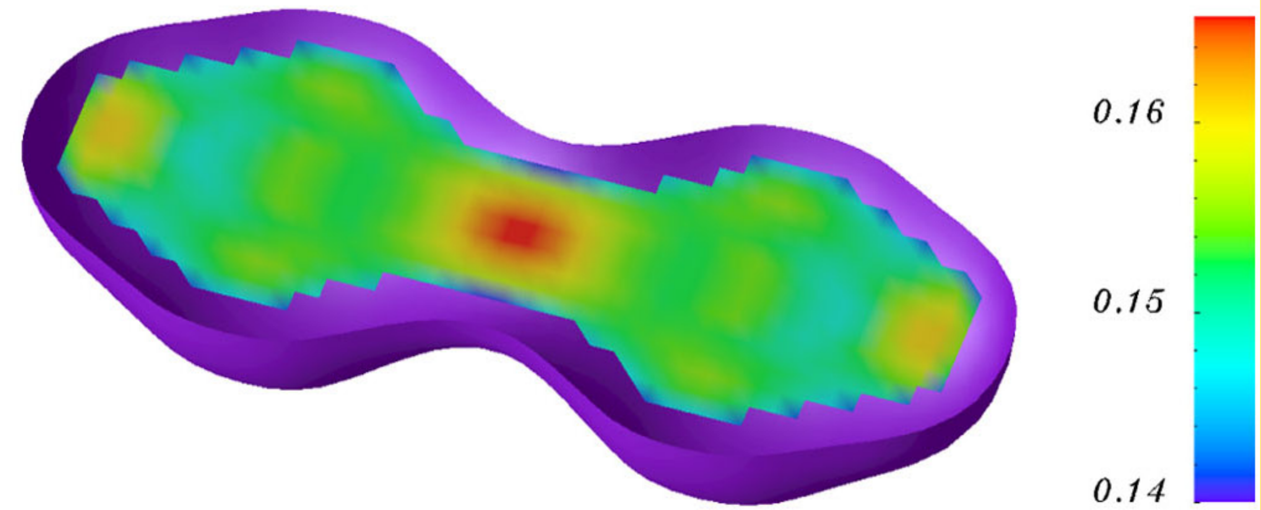

FIG. 3 (color online). Nucleon density (in fm^{-3}) in the colli- sion plane is plotted when the density in the neck reaches its maximum in the *xx* configuration at $E_{c.m.} = 900$ MeV. The half cut surface is an isodensity at $\rho_0/2$.

action plays a significant role is higher for such compact configurations.

At all energies, the *yx*, *yy*, and *yz* orientations exhibit roughly the same behavior, i.e., a rise and fall of T_{coll} with a maximum of $3-4 \times 10^{-21}$ s at $E_{c.m.} \sim 1200$ MeV. This

Collision calculation

- propagate all target eigenstates, $E_\nu < E_F$ and $E_\nu > E_F$
- inclusive e^+ spectrum: $\langle \bar{n}_q \rangle = \sum_{\nu > F} |a_{\nu,q}|^2$

$\sum$ all t_{in} e^- vacancies; how much flows into $E_q < -1$ at t_{f} ?

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restrict above sum to $\nu = 1S$

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random + true correlations

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random (small) + true correlations (most of it)

‘partial inclusive’ may overestimate true exclusive

Best shot at observing spontaneous pair process ?

exclusive [bound e^- & free e^+] pair vs inclusive process

E. Ackad and M.H.: PRA 78, 062711 (2008)
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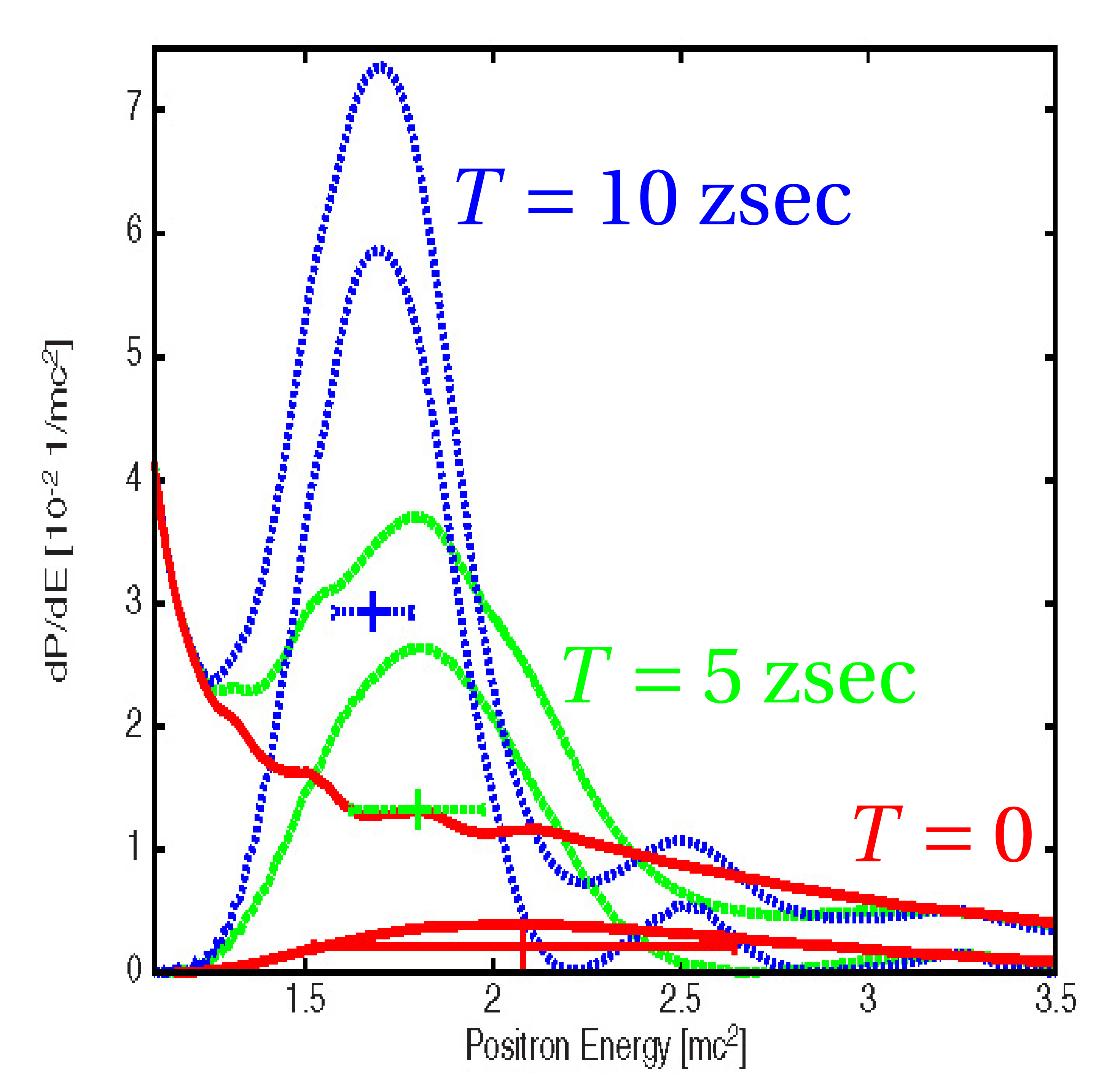

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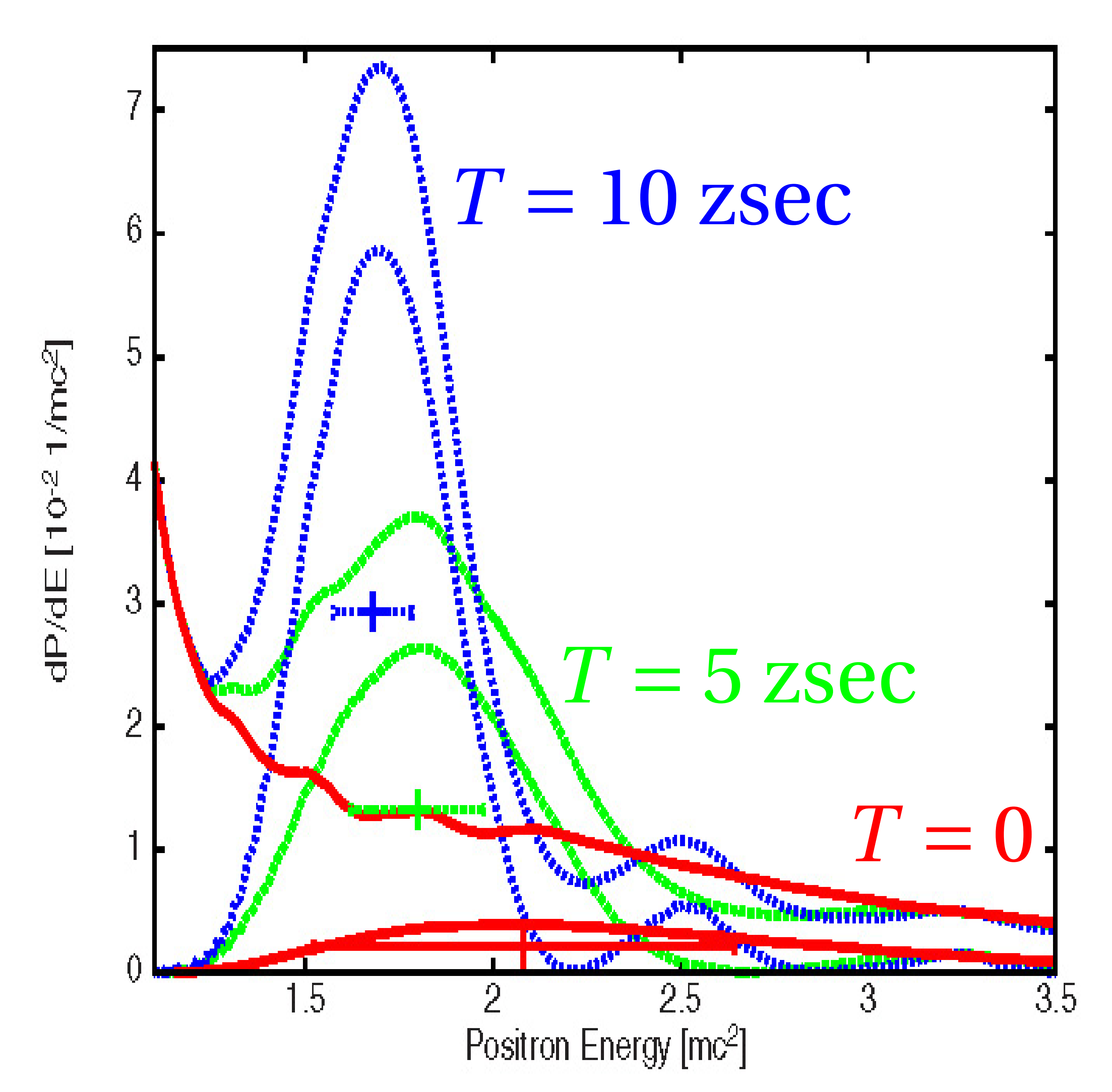

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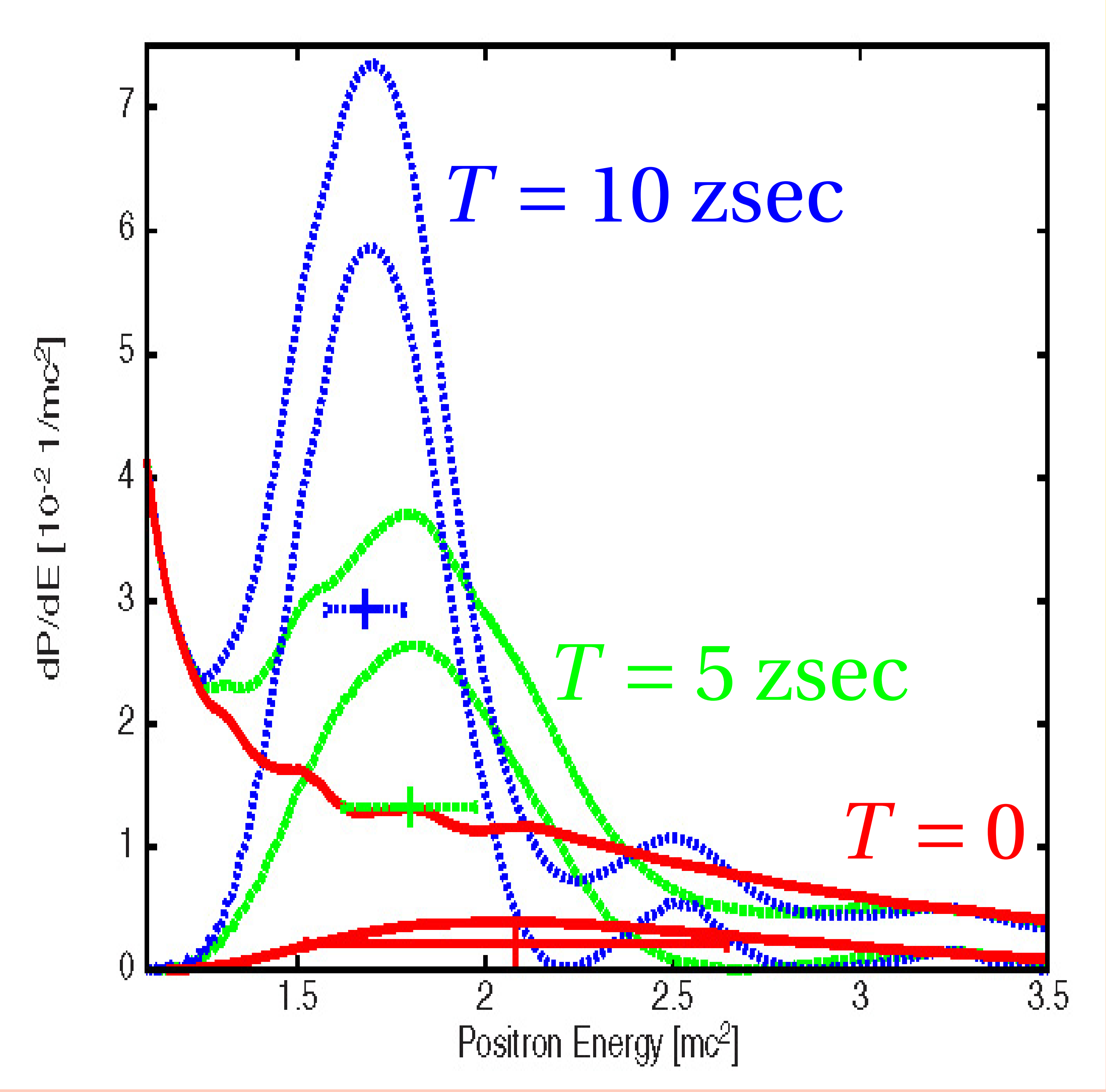
Width estimates:

$\frac{\hbar}{T+T_0}$ T_0 = diving time



1200 MeV U-U: TDHF predicts $T = 3.5 \times 10^{-21}$ sec

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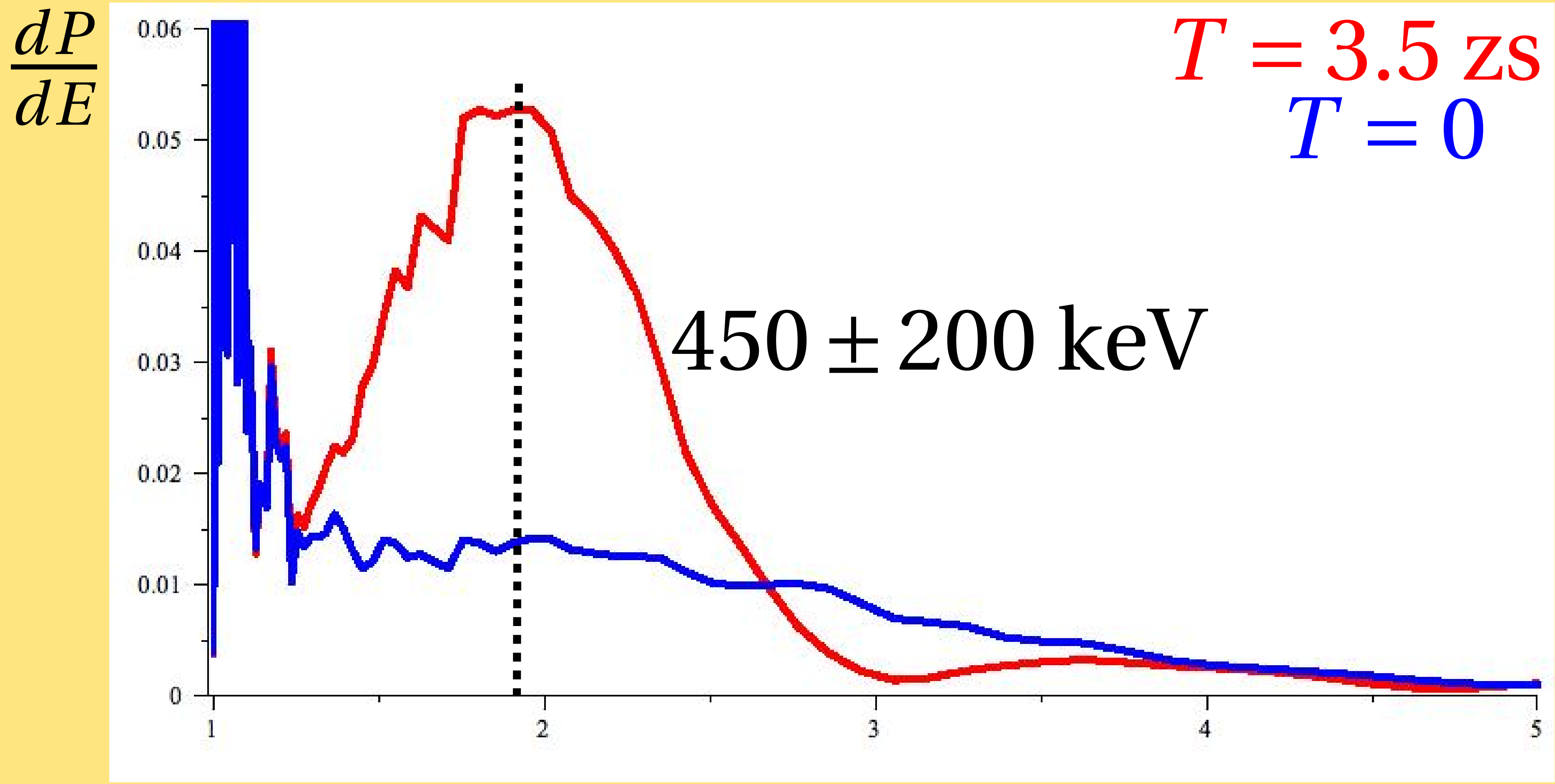

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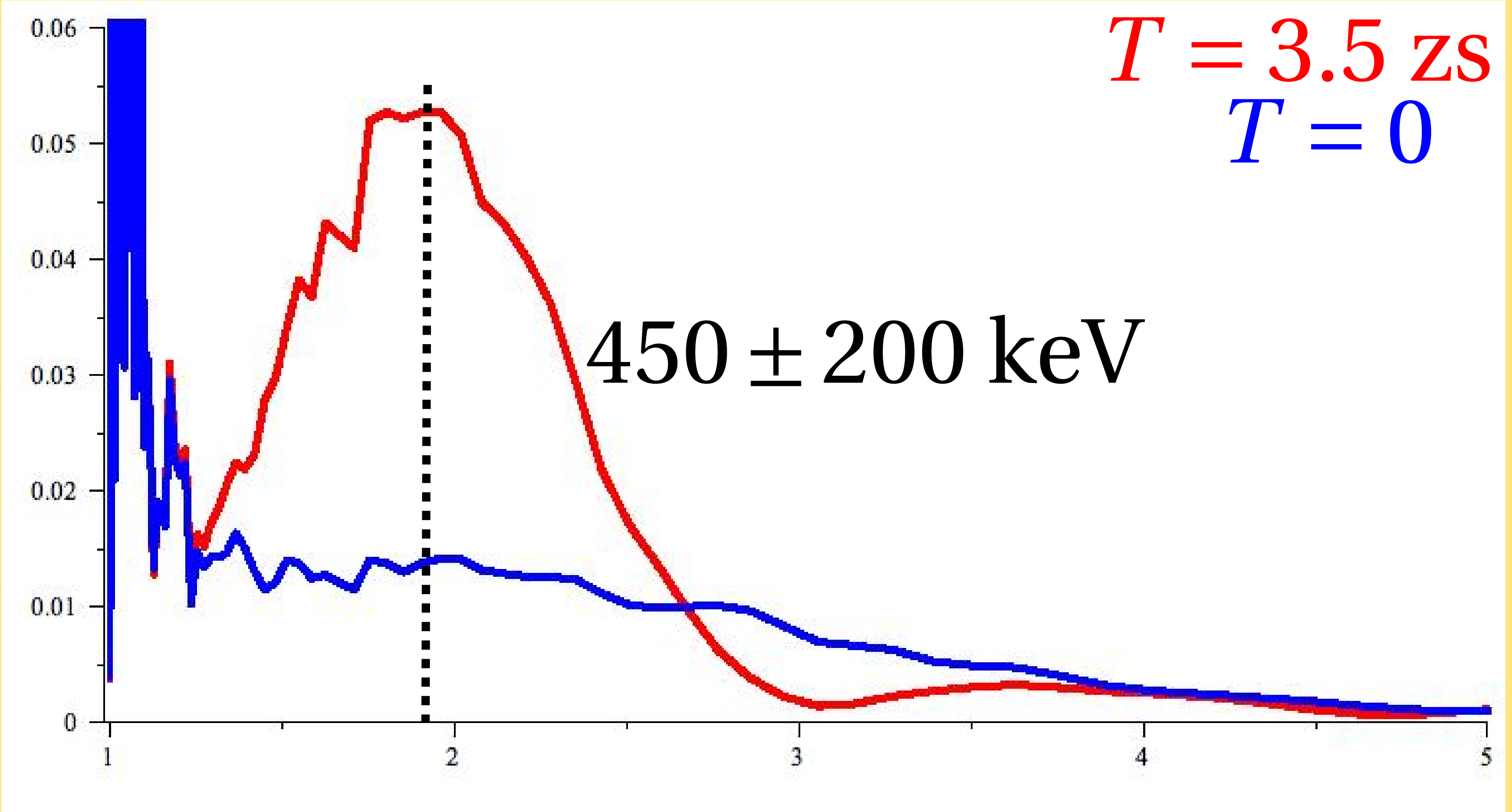

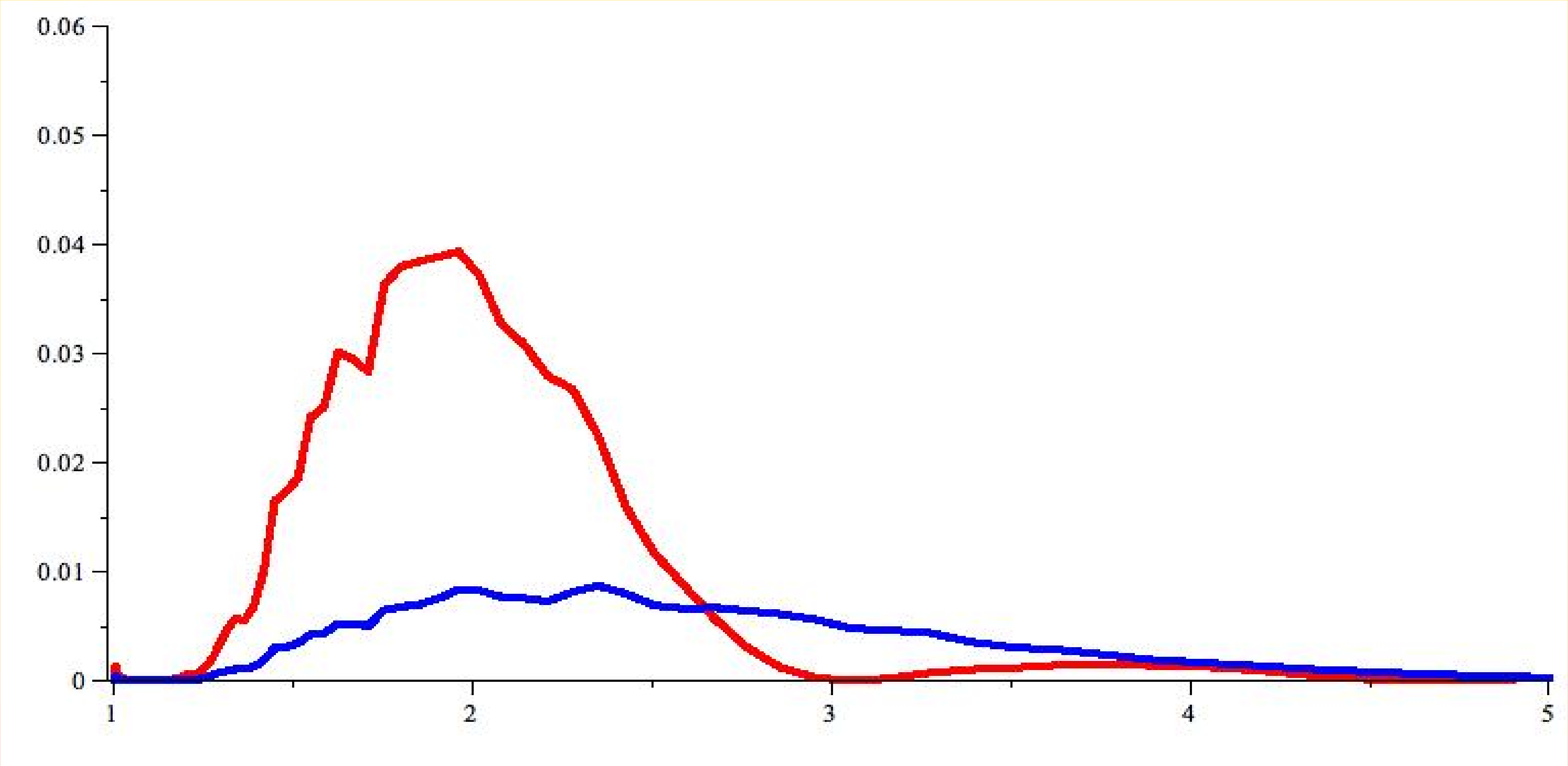

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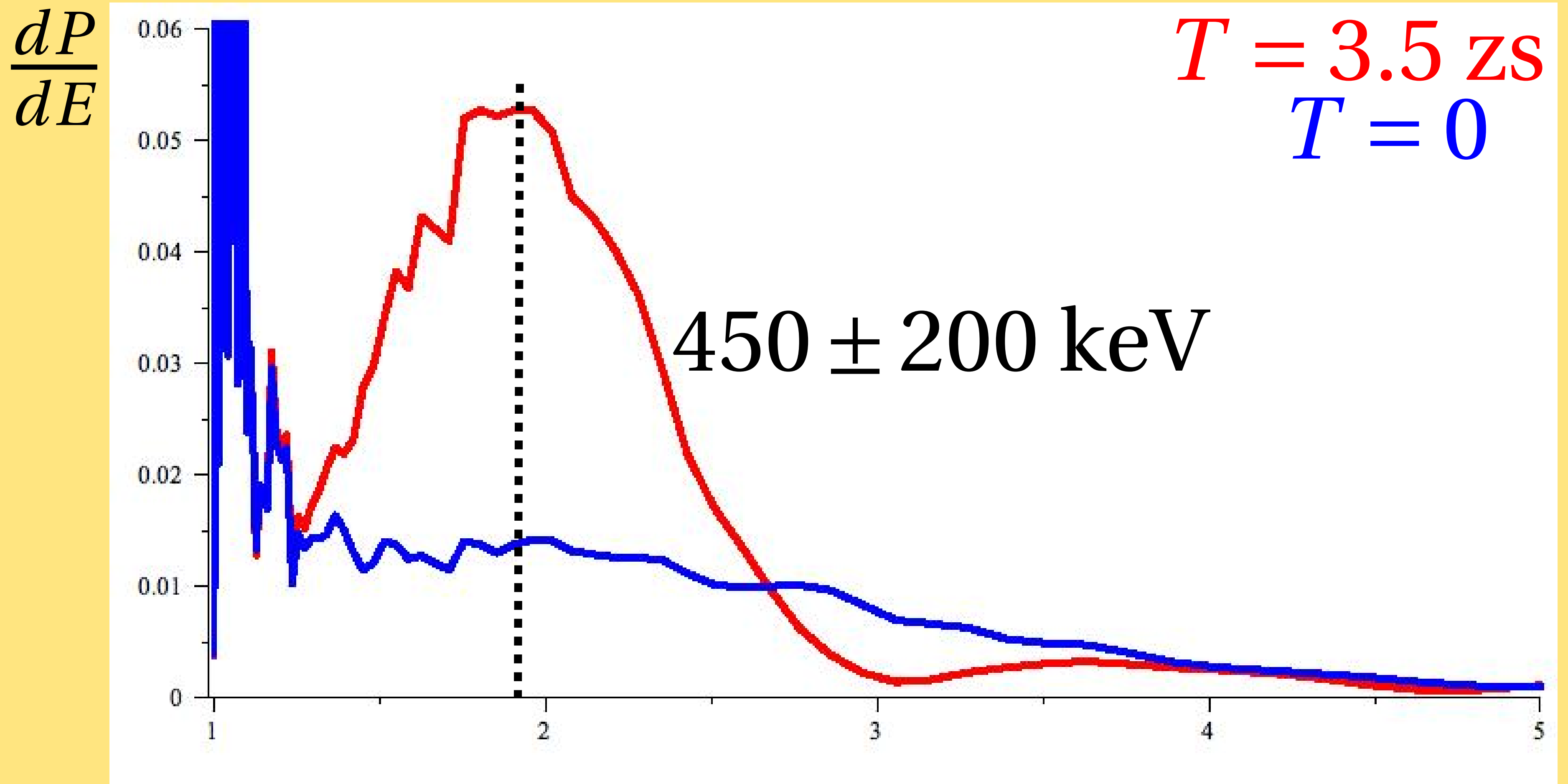
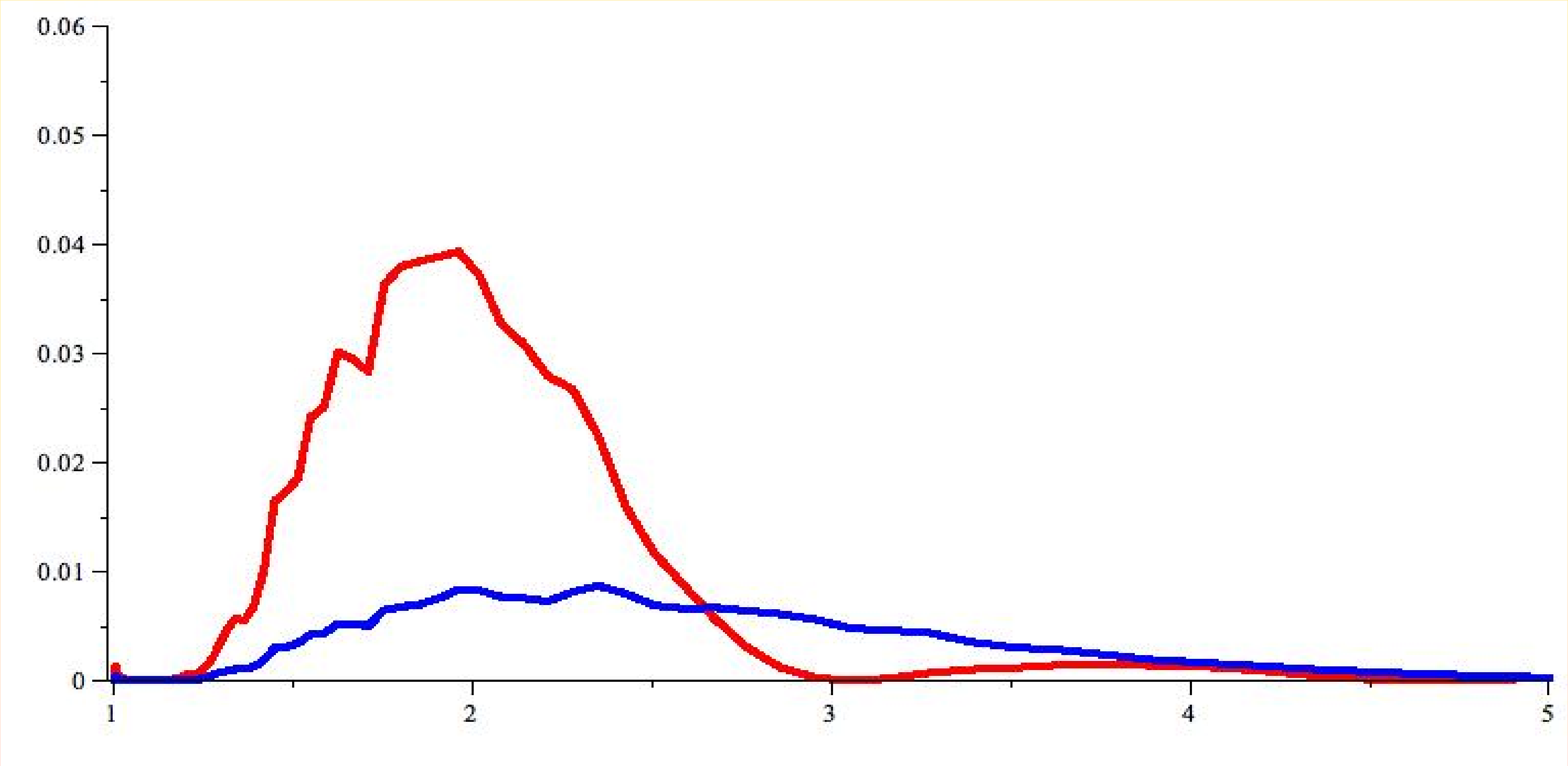
Why so broad ?





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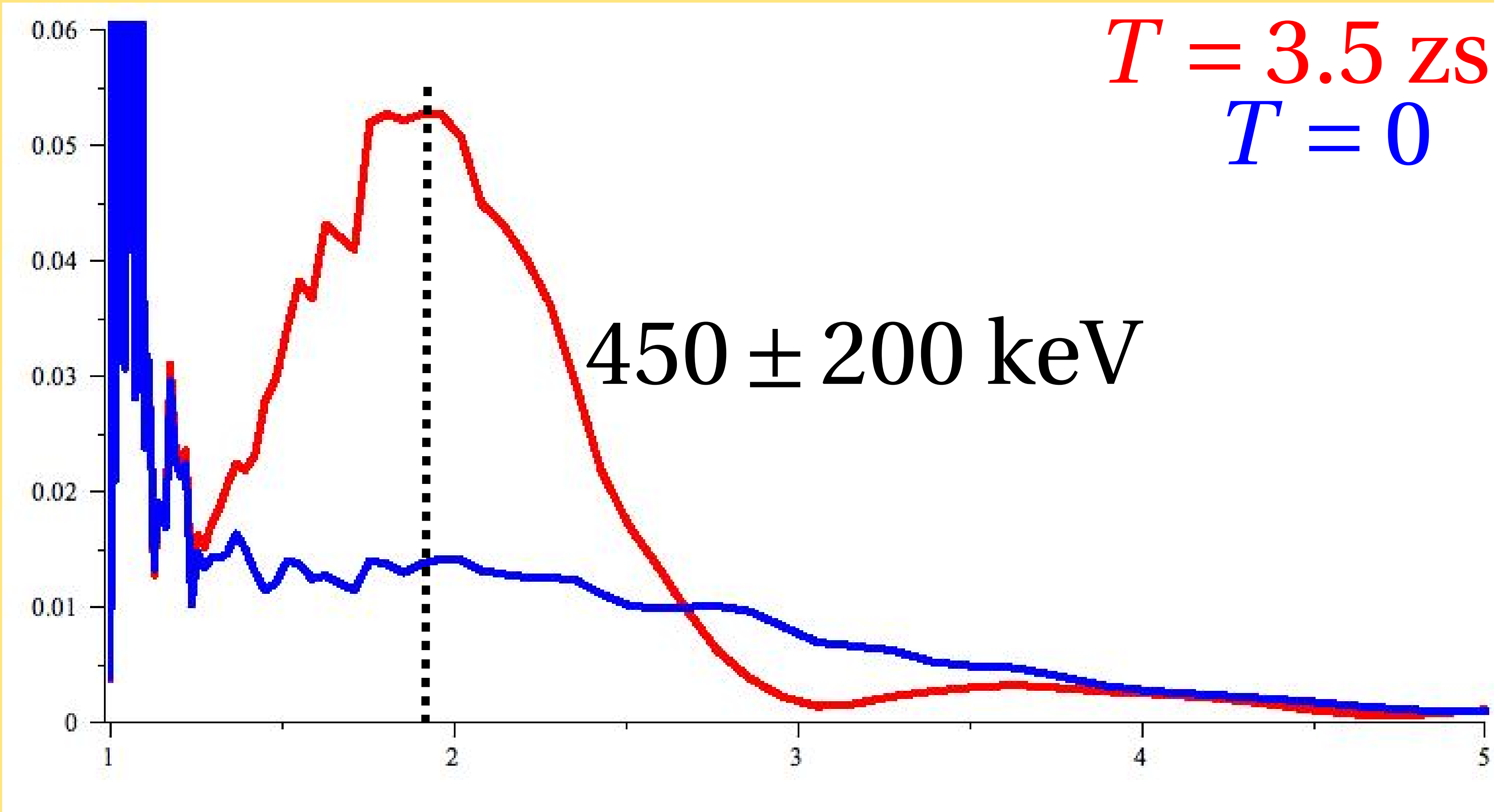
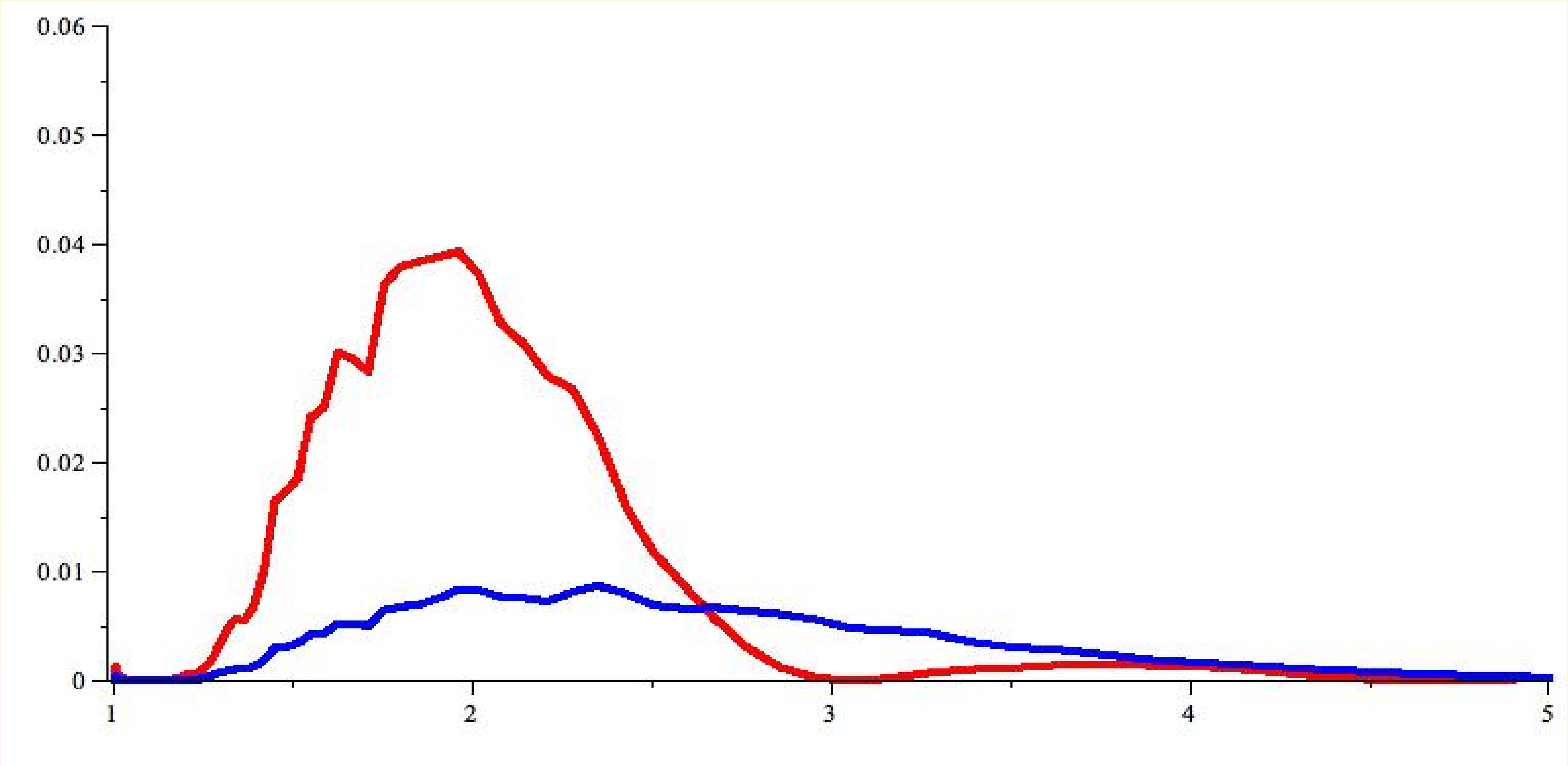
$\tau \approx 100\,T$

\+ dynamical BG

$\frac{dP}{dE}$





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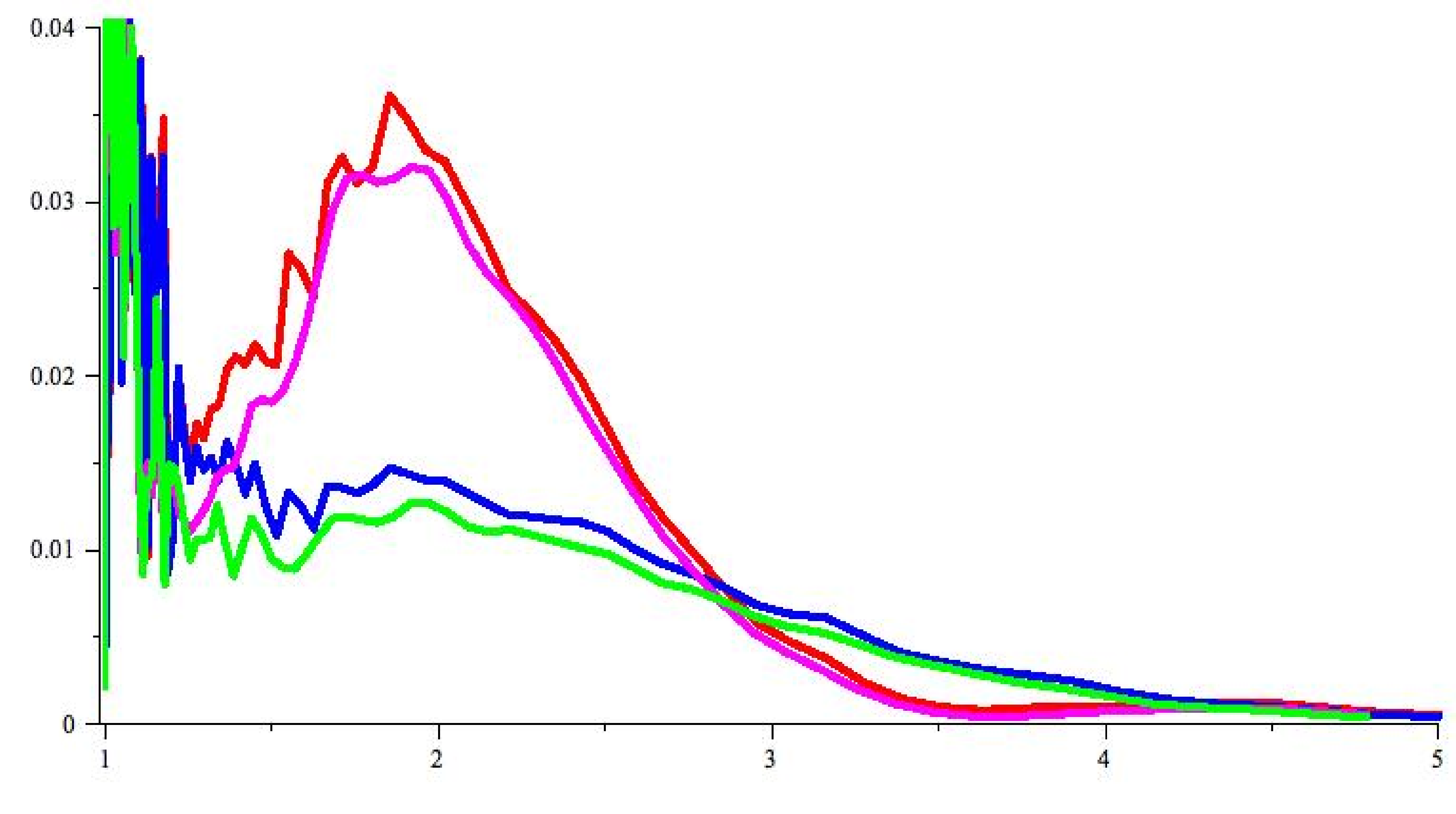
Detail: how accurate?

900 MeV U-U, $T = 0$ and $T = 2.5$ zs, inclusive
$N = 256$, two mFg basis parameter choices

$\frac{dP}{dE}$



Small-E noise: propagation of excited e^- vacancies is non-trivial

Large-N calculations are more stable

Conclusions

1) Pushing the envelope in resonance calculations

2) U^{92+} or U^{91+} collisions near $b = 0$ can do it

3) Nuclear theory: go to higher E_{CM}

will this raise the dynamical background? No!

4) exclusive [bound e^- & free e^+] spectrum is cleaner

experimental challenge ?

5) We calculated the inclusive e^+ spectrum by propagating *all* discretized continuum states