Hyperfine structure in heavy ions: towards a test of QED in strong fields

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Outline

Introduction and Motivation

Hyperfine structure in heavy ions

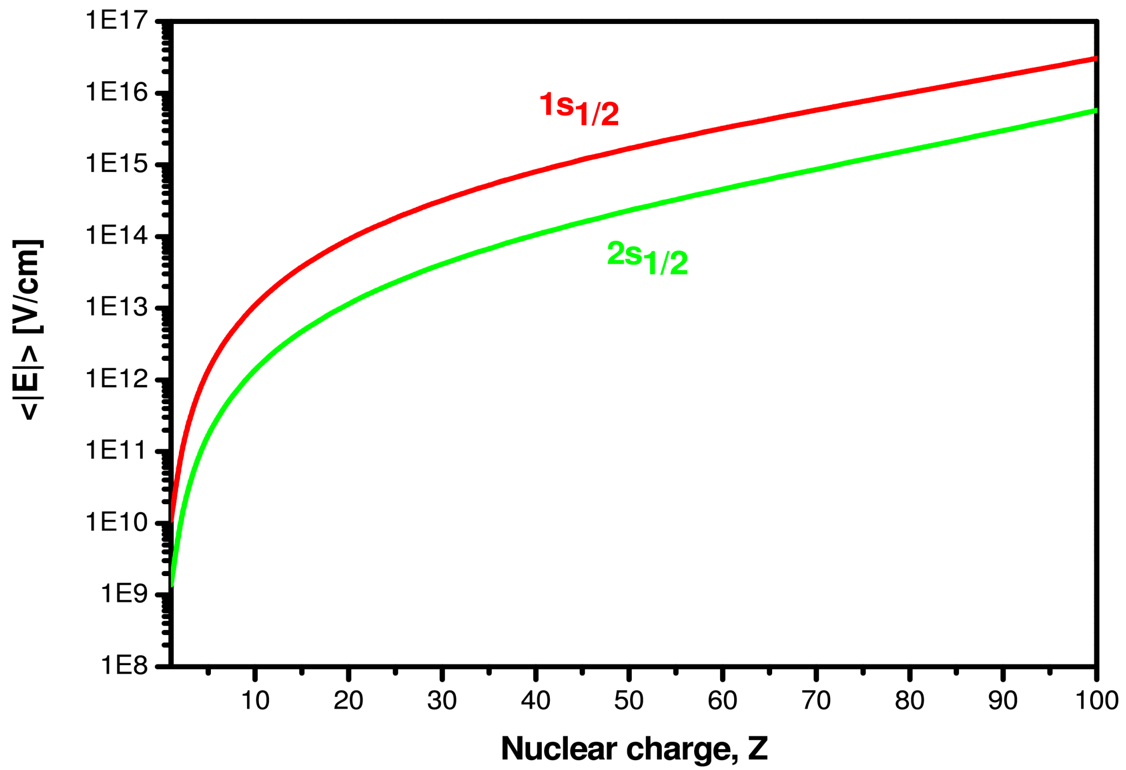
- hyperfine splittings in H- and Li-like heavy ions
- specific difference between H- and Li-like heavy ions
- screened QED corrections to hyperfine structure in Li-like ions

Numerical results

- specific difference between H- and Li-like Bi
- hyperfine splittings in Li- and B-like Bi

Conclusion and Outlook

Heavy few-electron ions provides possibility to test of QED at extremely strong electric fields



Interelectronic interaction ~ 1 / Z

QED ~ α

=> high-precision calculations are possible!

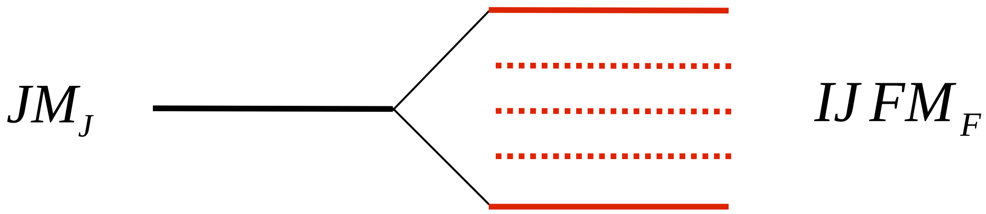
However, in contrast to light atoms, the parameter αZ is not small

In U^{92+}: $\alpha Z \approx 0.7$

=> test of QED to all orders in αZ

Hyperfine splitting in few-electron ions

$$F = I + J, \ldots, |I - J|$$



H-like ions $(1s)_{1/2}$

Li-like ions $(1s^2 2s)_{1/2}$

B-like ions $(1s^2 2s^2 2p)_{1/2}$

He-like ions $(1s^2)_0$

Be-like ions $(1s^2 2s^2)_0$

Introduction and Motivation

\# Investigations of the hyperfine structure in heavy ions provide

Fundamental physics

- high-precision test of the magnetic sector of bound-state QED in the nonperturbative regime (hyperfine splitting of H-, Li-, and B-like heavy ions)

  [Shabaev et al., PRL 2001]

Nuclear physics

- determination of the nuclear magnetic moments
- examination of nuclear models for the description of the Bohr-Weisskopf effect

Hyperfine structure in heavy ions

Measurements of the ground-state hyperfine splitting in H-like ions

Klaft et al., PRL 1994

$^{209}Bi^{82+}$ $\Delta E^{\rm exp} = 5.0840\,(8)$ eV

Crespo López-Urrutia et al., PRL 1996; PRA 1998

$^{165}Ho^{66+}$ $\Delta E^{\rm exp} = 2.1646\,(6)$ eV

$^{185}Re^{74+}$ $\Delta E^{\rm exp} = 2.7190\,(18)$ eV

$^{187}Re^{74+}$ $\Delta E^{\rm exp} = 2.7450\,(18)$ eV

Seelig et al., PRL 1998

$^{207}Pb^{81+}$ $\Delta E^{\rm exp} = 1.2159\,(2)$ eV

Beiersdorfer et al., PRA 2001

$^{203}Tl^{80+}$ $\Delta E^{\rm exp} = 3.21351\,(25)$ eV

$^{205}Tl^{80+}$ $\Delta E^{\rm exp} = 3.24409\,(29)$ eV

Hyperfine structure in heavy ions

Basic expression for the hyperfine splitting

$$\Delta E^{(a)} = \frac{\alpha(\alpha Z)^3}{n_a^3} \frac{g_I}{m_\mathrm{p}} \frac{2I+1}{(j_a+1)(2l_a+1)} \frac{1}{\left(1+\frac{m}{M}\right)^3}$$

$$\times \left[A(\alpha Z)(1-\delta)(1-\varepsilon) + \frac{B(\alpha Z)}{Z} + \frac{C(Z,\alpha Z)}{Z^2} + x_\mathrm{QED} + x_\mathrm{SQED} \right]$$

$A(\alpha Z)$ – relativistic factor

δ – nuclear charge distribution correction

ε – nuclear magnetization distribution correction

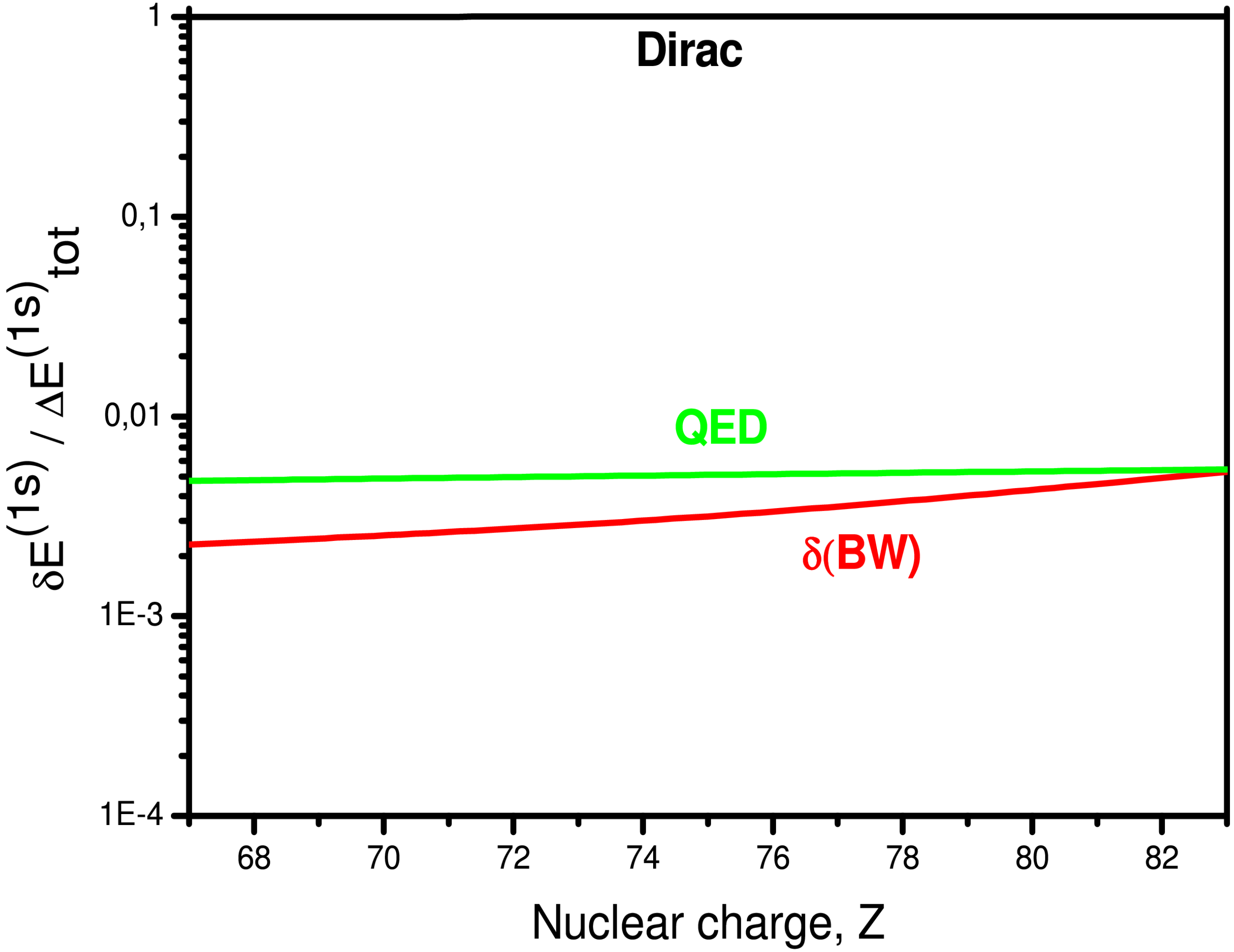
x_QED – one-electron QED correction

$B(\alpha Z)/Z$ – interelectronic-interaction correction of first-order in $1/Z$

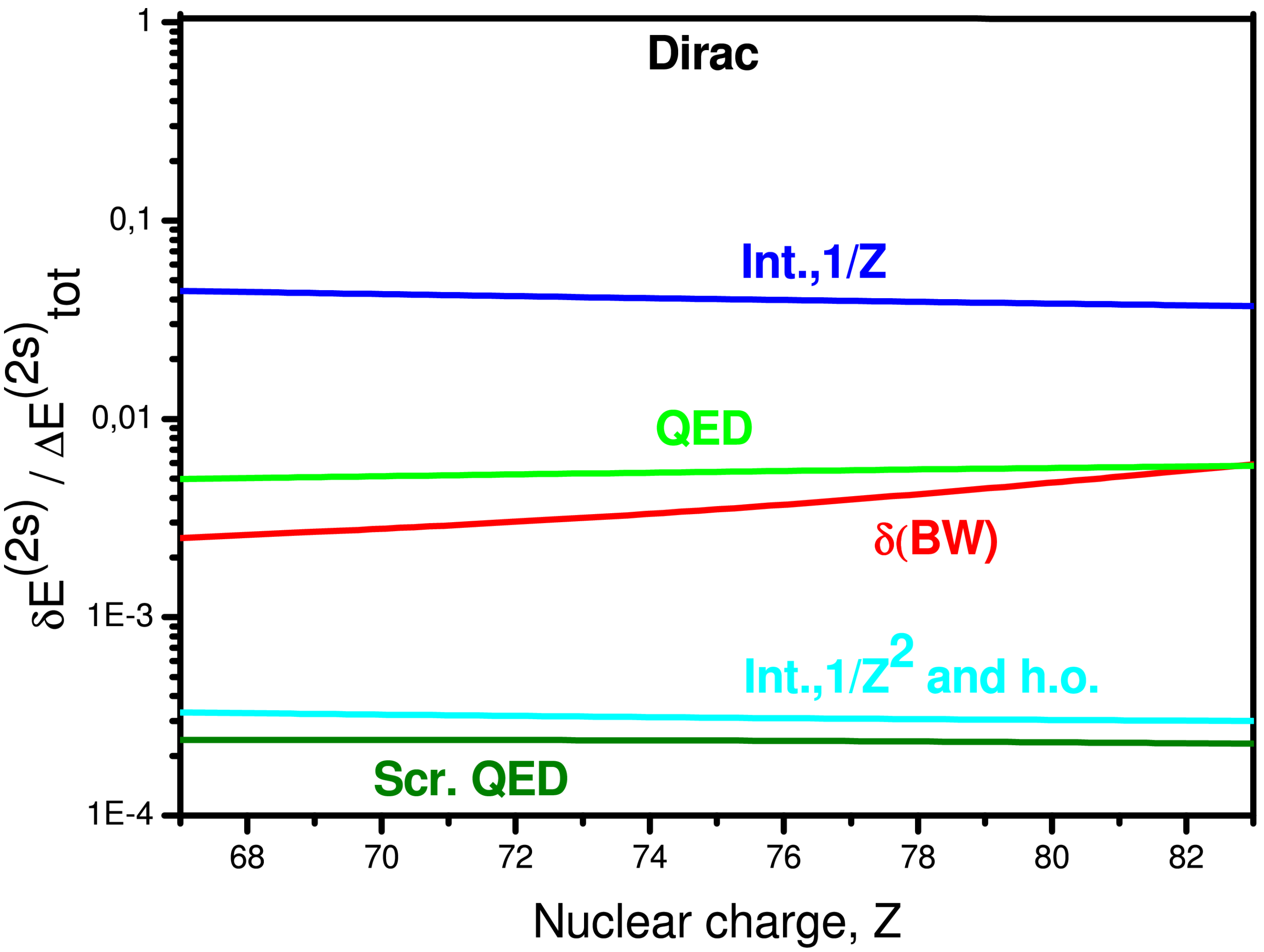
$C(Z,\alpha Z)/Z^2$ – $1/Z^2$ and higher-order interelectronic-interaction correction

x_SQED – screened QED correction

Ground-state hyperfine splitting in H-like ions



Ground-state hyperfine splitting in Li-like ions



Bohr-Weisskopf correction

Bohr-Weisskopf correction depends linearly on the functions $K_S(r)$ and $K_L(r)$

$$\varepsilon \sim K_S(r), K_L(r)$$

$$K_S(r) = \frac{\int_0^r g(r') f(r') dr'}{\int_0^\infty g(r') f(r') dr'}, \qquad K_L(r) = \frac{\int_0^r \left(1 - \frac{r'^3}{r^3}\right) g(r') f(r') dr'}{\int_0^\infty g(r') f(r') dr'}$$

[Shabaev et al., PRA 1998]

Hyperfine structure in heavy ions

Bohr-Weisskopf correction

For a given κ the radial Dirac equations are the same in the nuclear region

$$\left[-i\sigma_y \frac{d}{dr} + \sigma_x \frac{\kappa}{r} + \sigma_z m - \varepsilon + V_{\mathrm{nucl}}(r)\right]\begin{pmatrix} rg \\ rf \end{pmatrix} = 0$$

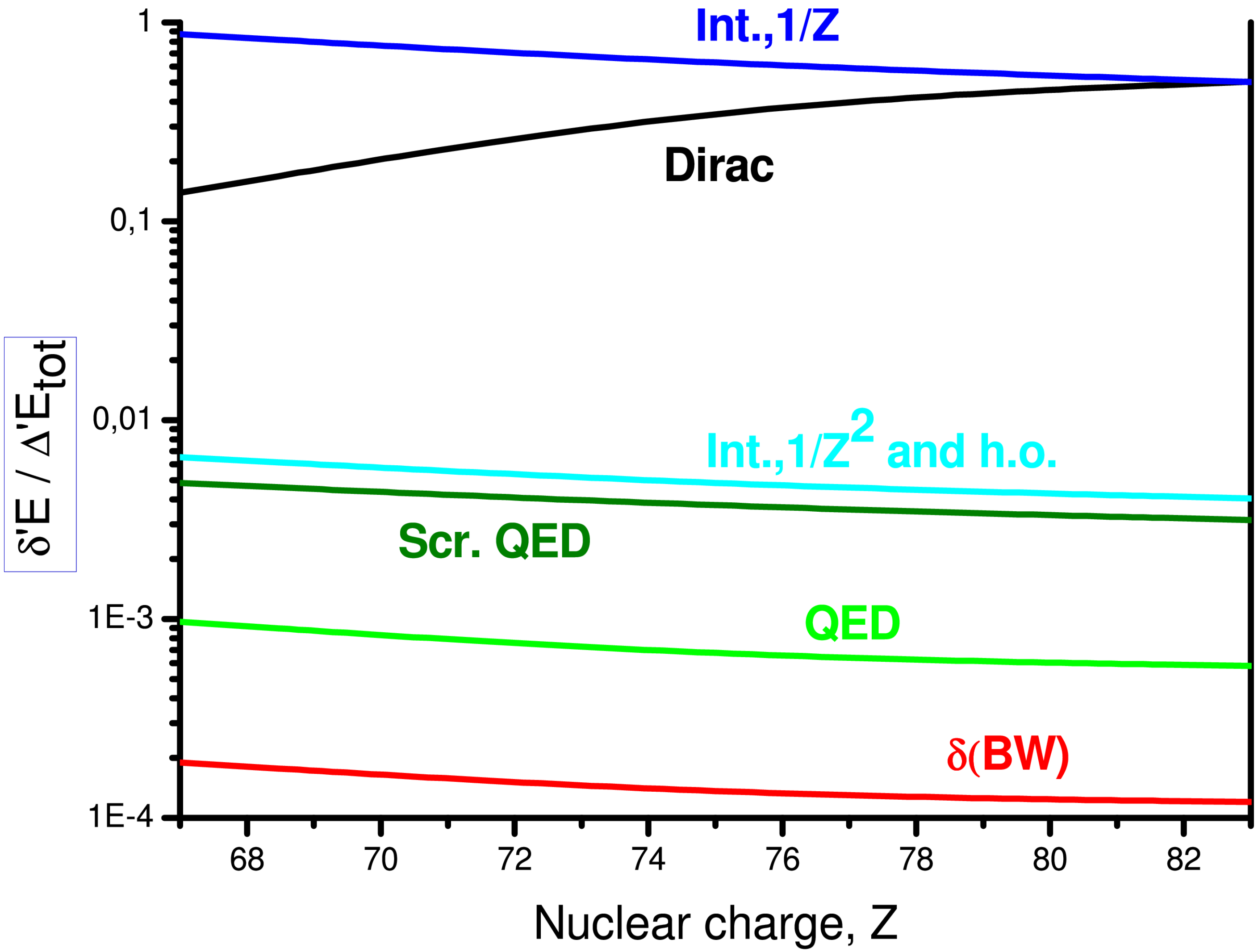
$$\varepsilon_a \approx \frac{(\alpha Z)^2}{2n_a^2} << \frac{\alpha Z}{R_{\mathrm{nucl}}} = \left|V_{\mathrm{nucl}}(R_{\mathrm{nucl}})\right|$$

=> the ratio of the Bohr-Weisskopf corrections is very stable with respect to variations of the nuclear models

$$\frac{K_S^{(2s)}(r)}{K_S^{(1s)}(r)} = \frac{K_L^{(2s)}(r)}{K_L^{(1s)}(r)} = \frac{\varepsilon^{(2s)}}{\varepsilon^{(1s)}} = f(\alpha Z)$$

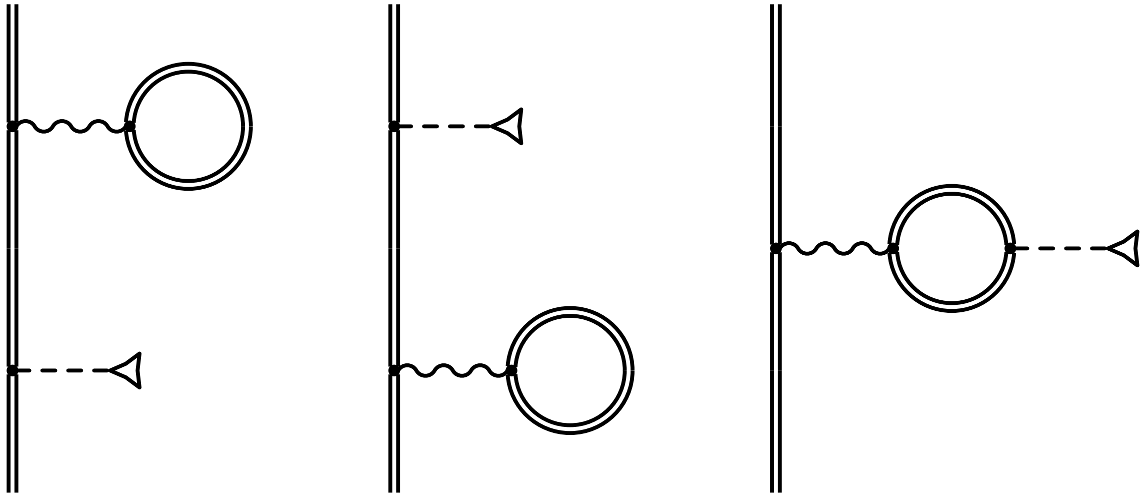
[Shabaev et al., PRL 2001]

$$\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)}$$



Vacuum-polarization correction

Second-order terms in perturbation theory expansion



[Schneider, Greiner, and Soff, PRA 1994]

[Sunnergren, Persson, Salomonson, Schneider, Lindgren, and Soff, PRA 1998]

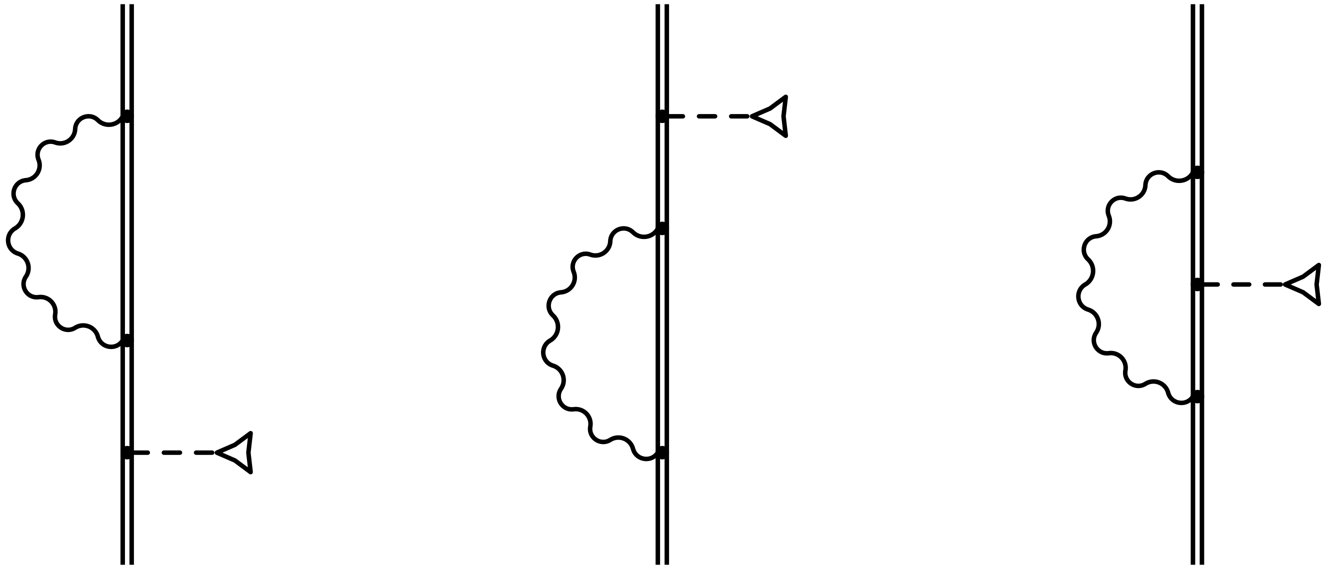
[Artemyev, Shabaev, Plunien, Soff, and Yerokhin, PRA 2001]

[Sapirstein and Cheng, PRA 2001]

Hyperfine structure in heavy ions

Self-energy correction

Second-order terms in perturbation theory expansion



[Persson, Schneider, Greiner, Soff, and Lindgren, PRL 1996]

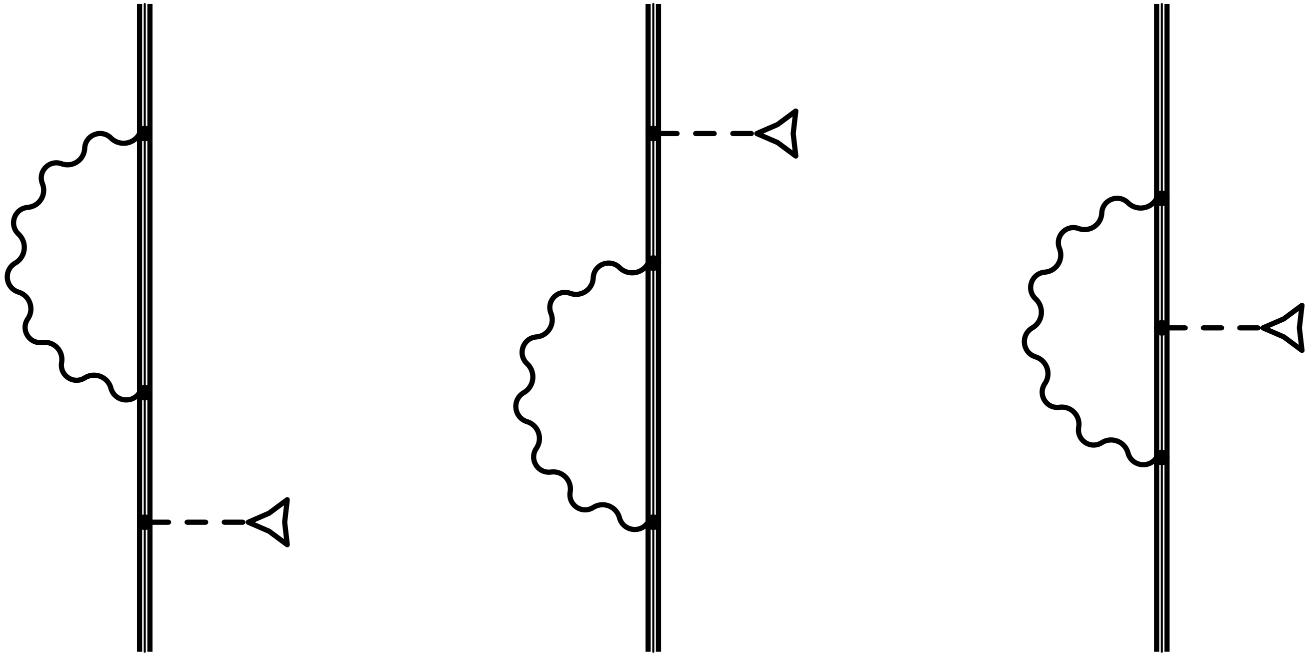
[Blundell, Cheng, and Sapirstein, PRA 1997]

[Shabaev, Tomaselli, Kühl, Artemyev, and Yerokhin, PRA 1997]

[Yerokhin and Shabaev, PRA 2001]

Screened self-energy correction: effective potential approach

$$V_{\rm nucl}(r) \rightarrow V_{\rm eff}(r) = V_{\rm nucl}(r) + V_{\rm scr}(r)$$



Screened self-energy correction: effective potential approach

Different screening potential have been employed

- core-Hartree potential

$$V_{\rm scr}(r) = \alpha \int_0^\infty \frac{\rho_c(r')}{r_>} dr'$$

$\rho_c(r)$ – density of the core electrons

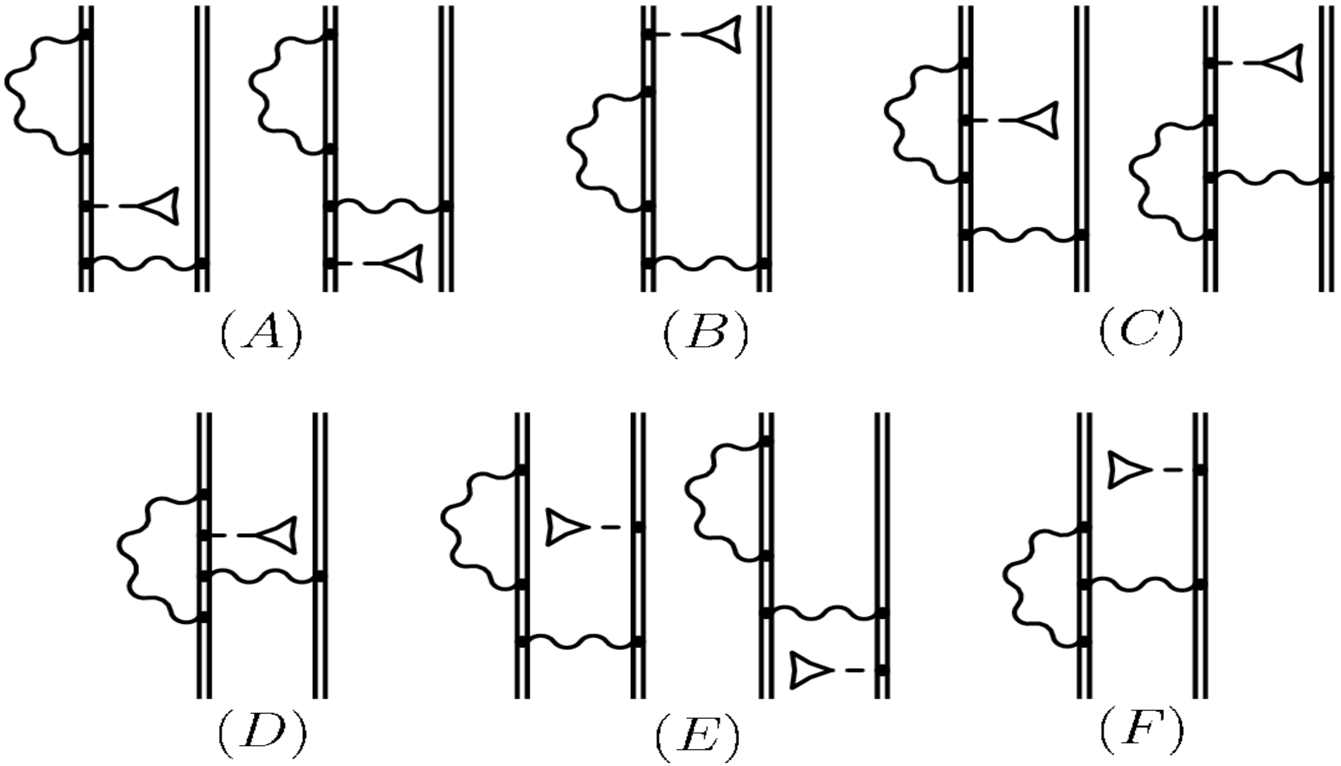
- Kohn-Sham potential

$$V_{\rm scr}(r) = \alpha \int_0^\infty \frac{\rho_t(r')}{r_>} dr' - \frac{2}{3}\frac{\alpha}{r}\left[\frac{81}{32\pi^2} r\rho_t(r)\right]^{1/3}$$

$\rho_t(r)$ – total electron density

Screened self-energy correction: rigorous evaluation

Third-order terms in perturbation theory expansion



36 diagrams

Screened self-energy correction: rigorous evaluation

Ultraviolet divergences: diagrams (A), (B), (C), (E), and (F)

Divergent zero- and one potential terms in diagrams (A), (B), and (E) and zero-potential terms in diagrams (C) and (F) are separated out and calculated in momentum space.

Infrared divergences: diagrams (C), (D), and (F)

Divergences are regularized by introducing a nonzero photon mass and cancelled analytically.

Numerical results

Screened self-energy correction x_{SQED}(SE) in the Feynman and Coulomb gauges for the Li-like $^{209}Bi^{80+}$

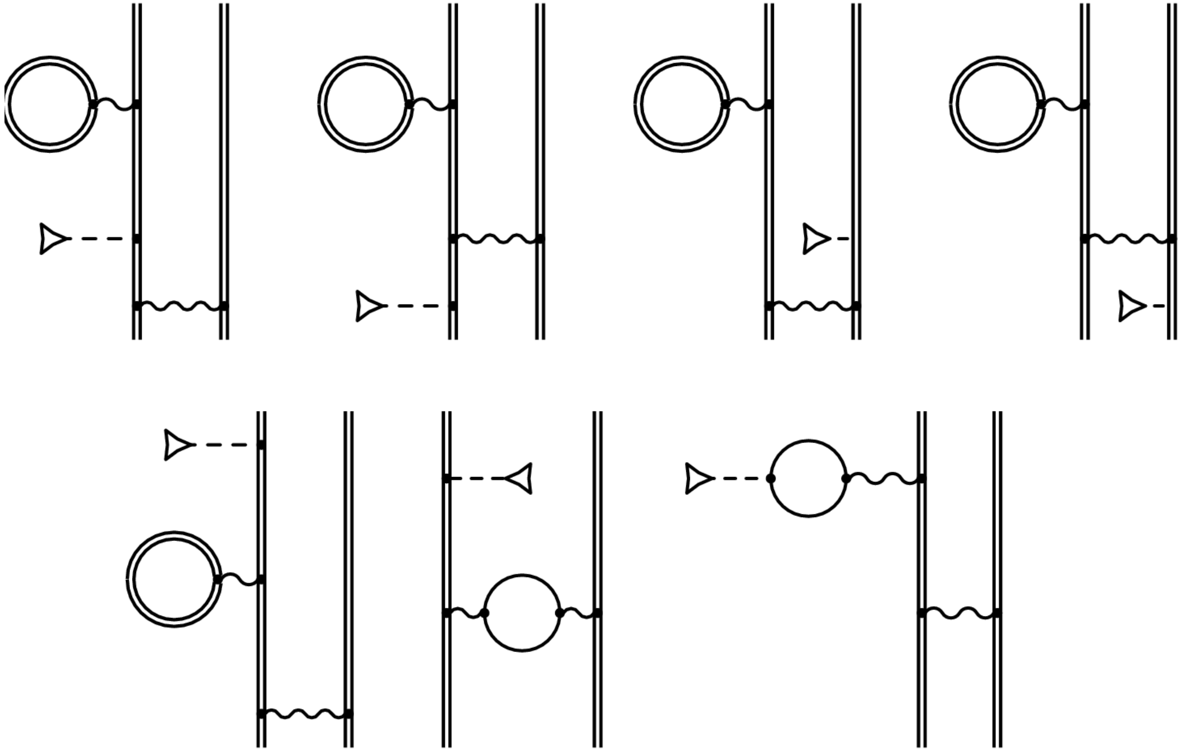
	Feynman	Coulomb
A, irr	0.001544	0.001555
B, irr	-0.000380	-0.000398
C, irr	0.001928	0.001952
D, irr	-0.000936	-0.000945
E, irr	0.000028	0.000028
F, irr	-0.000174	-0.000172
G, red	-0.001298	-0.001307
H, red	0.000331	0.000331
I, red	0.000066	0.000066
Total	0.001109	0.001109

Kohn-Sham screening	0.0012
core-Hartree screening	0.0013

Hyperfine structure in heavy ions

Screened vacuum-polarization correction: rigorous calculation

Third-order terms in perturbation theory expansion



28 diagrams

Numerical results

Specific difference between hyperfine splitting in H- and Li-like bismuth in meV

$$\Delta' E = \Delta E^{(2s)} - \xi \Delta E^{(1s)};$$ for Z=83 we obtain ξ=0.16886

	$\xi \Delta E\,(1s)$	$\Delta E\,(2s)$	$\Delta' E$
Dirac value	876.638	844.829	-31.809
QED	-5.088	-5.052	0.036
Screened QED		0.194(6)	0.194(6)
local potential approx.		0.21(4)	0.21(4)
Interel. int. 1/Z		-29.995	-29.995
Interel. int. higher orders		0.25(4)	0.25(4)
Total theory:			-61.32(4)

Remaining uncertainty ≈ 0.005 – 0.010 meV

=> possibility for a test of screened QED on the level of few percent

[Volotka, Glazov, Shabaev, Tupitsyn, and Plunien, PRL 2009]

Numerical results

Bohr-Weisskopf corrections for H-, Li-, and B-like bismuth

Knowing 1s hyperfine splitting from experiment, the Bohr-Weisskopf correction can be obtained

$$\varepsilon^{(1s)} = \frac{\Delta E_{\mathrm{NS}}^{(1s)} + \Delta E_{\mathrm{QED}}^{(1s)} - \Delta E_{\mathrm{exp}}^{(1s)}}{\Delta E_{\mathrm{NS}}^{(1s)}} = 0.0148(5)$$

The ratio of the Bohr-Weisskopf corrections

$$\frac{\varepsilon^{(2s)}}{\varepsilon^{(1s)}} = 1.0782(3) \qquad \frac{\varepsilon^{(2p)}}{\varepsilon^{(1s)}} = 0.295(2)$$

Numerical results

Hyperfine splitting in Li- and B-like bismuth in meV

	ΔE(2s)	ΔE(2p)
Dirac value (point nucl.)	958.5	296.35
Nuclear size	-113.7(6)	-9.84(5)
Bohr-Weisskopf	-13.1(6)	-1.11(5)
Interelectronic interaction	-29.7	-27.31(22)
QED	-4.8	-0.25(2)
Total theory:	797.22(15)	257.83(22)
Exp.:	820(26)*	
	791(5)**	

*Beiersdorfer et al., PRL 1998

**Beiersdorfer et al., unpublished

Summary

Conclusion

- rigorous evaluation of the complete gauge-invariant set of the screened QED corrections has been performed
- the most accurate theoretical prediction for the specific difference between hyperfine structure values in H- and Li-like Bi has been obtained

Outlook

- *ab initio* calculation of the second order interelectronic-interaction correction
- rigorous evaluation of the screened vacuum-polarization (Wichmann-Kroll part) correction